6.5 Part 1 The Law of Sines

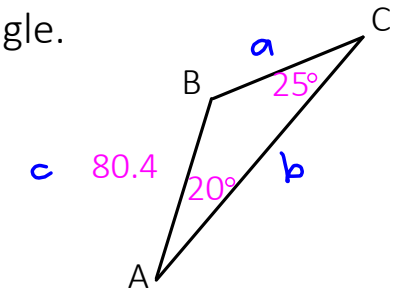
The Law of Sines is used to solve triangles when given ASA or AAS.
Tomorrow we will discuss how to solve when given SSA.

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

where A, B, and C are the angles and a, b, and c are the sides.

Example 1: Solve the triangle.



$m\angle A = 20°$
$m\angle B = 135°$
$m\angle C = 25°$
$AB = 80.4$
$BC \approx 65.07$
$AC \approx 134.52$

$\frac{\sin 25°}{80.4} = \frac{\sin 20°}{a}$

$\frac{\sin 25°}{80.4} = \frac{\sin 135°}{b}$

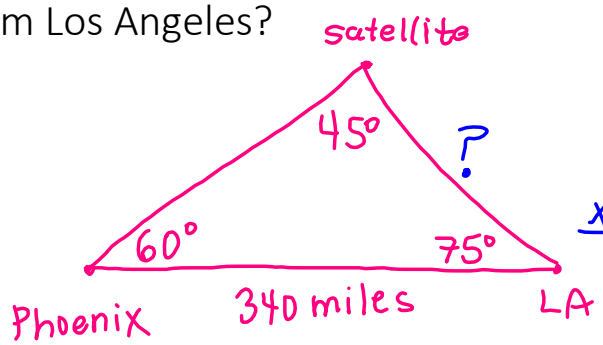
$\frac{a \sin 25°}{\sin 25°} = \frac{80.4 \sin 20°}{\sin 25°}$

$a \approx 65.07$

$\frac{b \sin 25°}{\sin 25°} = \frac{80.4 \sin 135°}{\sin 25°}$

$b \approx 134.52$

Example 2: A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 miles apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 60°at Phoenix and 75°at Los Angeles. How far is the satellite from Los Angeles?



$$\frac{\sin 45^\circ}{340} = \frac{\sin 60^\circ}{X}$$

$$\frac{X \cancel{\sin 45^\circ}}{\cancel{\sin 45^\circ}} = \frac{340 \sin 60^\circ}{\sin 45^\circ}$$

$X \approx 416.41$ miles

Example 3: Given A = 42°, B = 35°, and a = 10, find b.

$$\frac{\sin 42^\circ}{10} = \frac{\sin 35^\circ}{b}$$

$$b = \frac{10 \sin 35^\circ}{\sin 42^\circ}$$

$b \approx 8.57$

Example 4: Given A = 5°, C = 25°, and a = 15, find c.

$$\frac{\sin 5^\circ}{15} = \frac{\sin 25^\circ}{c}$$

$$c = \frac{15 \sin 25^\circ}{\sin 5^\circ}$$

$c \approx 72.74$

Example 5: Solve each triangle.

a) A = 43°, B = 52°, b = 20, C = 85°, a ≈ 17.31, c ≈ 25.28

$\frac{\sin 43°}{a} = \frac{\sin 52°}{20}$

$a = \frac{20 \sin 43°}{\sin 52°}$

$a \approx 17.31$

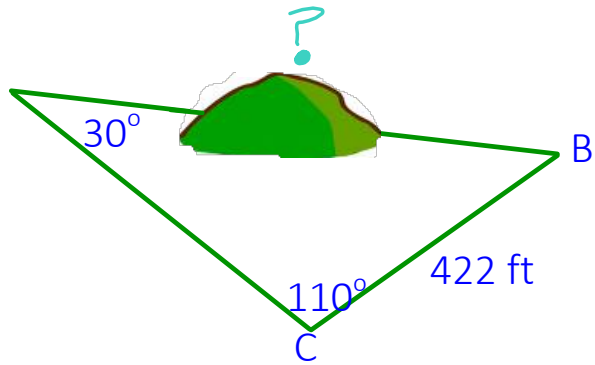
$\frac{\sin 52°}{20} = \frac{\sin 85°}{c}$

$c = \frac{20 \sin 85°}{\sin 52°}$

$c \approx 25.28$

b) A = 23°, B = 62°, c = 15

Example 6: A surveying crew needs to find the distance between two points. They cannot measure the distance directly because there is a hill between the two points. The surveyors obtain the information shown in the diagram. Find the distance.



$\frac{\sin 110°}{c} = \frac{\sin 30°}{422}$

$c = \frac{422 \sin 110°}{\sin 30°}$

$c \approx 793.10$ ft

Example 7: Two rangers, one at station A and one at station B, observe a fire in the forest. The angle at station A formed by the lines of sight to station B and to the fire is 65.23°. The angle at station B formed by the lines of sight to station A and to the fire is 56.47°. The stations are 10 km apart.

a) How far from station A is the fire?

b) How far from station B is the fire?

a) $\frac{\sin 58.3^\circ}{10} = \frac{\sin 56.47^\circ}{x}$

9.80 km

b) $\frac{\sin 65.23^\circ}{y} = \frac{\sin 58.3^\circ}{10}$

10.67 km