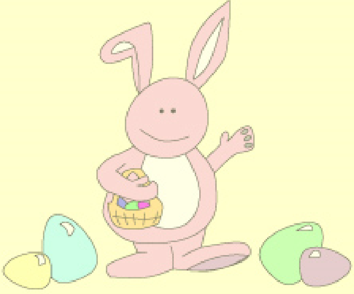
7.1 The Pythagorean Theorem

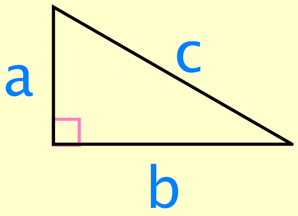
Theorem 7.1: The Pythagorean Theorem

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

In other words...

If it is a right triangle, then $a^2 + b^2 = c^2$.





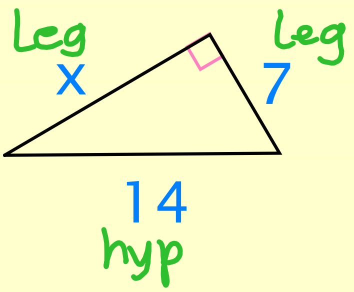
A Pythagorean triple is a group of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the greatest number.

One common Pythagorean triple is 3, 4, & 5 and another is 5, 12, & 13.

If the measures of the sides of a right triangle are whole numbers, then the measures are a Pythagorean triple.

Example 1

Find the length of the leg of the right triangle.



$x^2 + 7^2 = 14^2$

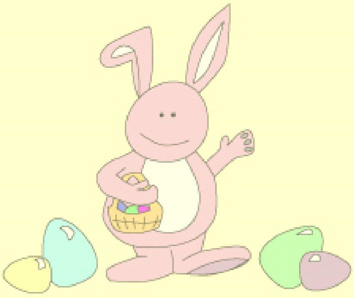
$\cancel{x^2 + 49} = 196$

$-\cancel{49} \quad -49$

$\sqrt{x^2} = \sqrt{147}$

$\boxed{x = 7\sqrt{3}}$

7 | 147
3 | 21
7



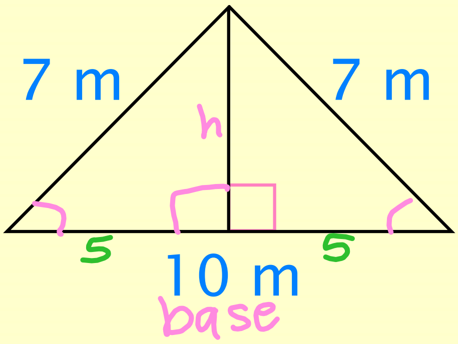
Example 2

Find the area of the triangle.

$A = \frac{bh}{2}$ or $\frac{1}{2}bh$

$A = \frac{(10)(\cancel{2}\sqrt{6})}{\cancel{2}}$

$\boxed{A = 10\sqrt{6}\ m^2}$



$5^2 + h^2 = 7^2$

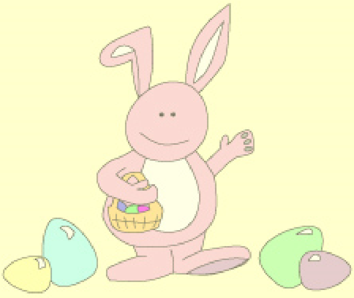
$25 + h^2 = 49$

$-25 \quad -25$

$\sqrt{h^2} = \sqrt{24}$

$h = 2\sqrt{6}$

2 | 24
2 | 12
2 | 6
3



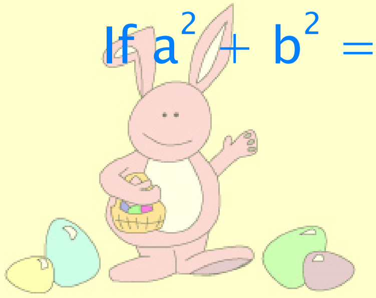
7.2 The Converse of the Pythagorean Theorem

Theorem 7.2: Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

In other words...

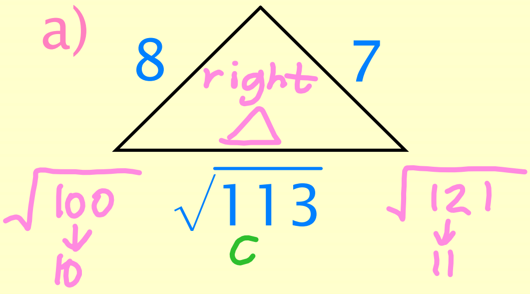
If $a^2 + b^2 = c^2$, then it is a right triangle.



Example 3

The triangles below appear to be right triangles. Determine whether or not this is so.

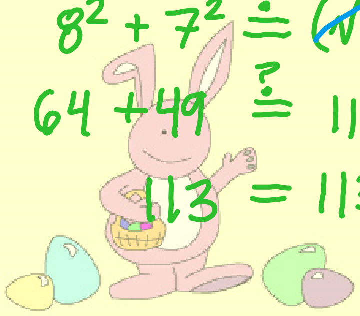
a)



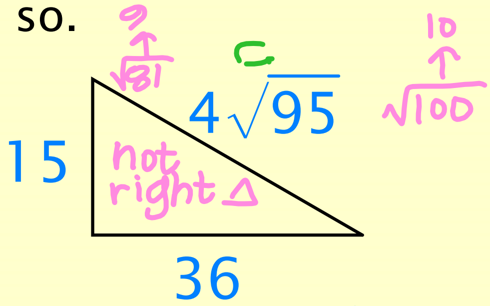
$$8^2 + 7^2 \overset{?}{=} (\cancel{\sqrt{113}})^{\cancel{2}}$$

$$64 + 49 \overset{?}{=} 113$$

$$113 = 113\checkmark$$



b)



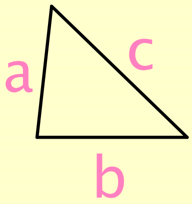
$$15^2 + 36^2 \overset{?}{=} (4\sqrt{95})^2$$

$$225 + 1296 \overset{?}{=} 16 \cdot 95$$

$$1521 \overset{?}{\neq} 1520$$

Theorem 7.3

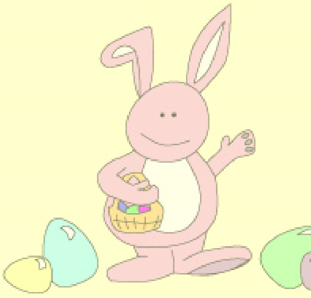
If $c^2 < a^2 + b^2$, then the triangle is acute.

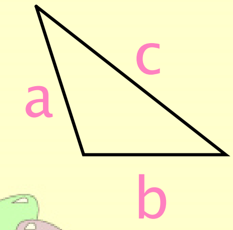


$$c^2 < a^2 + b^2$$

Theorem 7.4

If $c^2 > a^2 + b^2$, then the triangle is obtuse.





$$c^2 > a^2 + b^2$$

Example 4

$c^2 \boxed{}\ a^2 + b^2$

Determine if a triangle can be formed by the given side lengths. If so, classify the triangle.

a) 9, 10, 15 (c)

✓ $9+10>15$
✓ $10+15>9$
✓ $15+9>10$
yes △

$15^2 \boxed{}\ 9^2+10^2$

$225 \boxed{}\ 81+100$

$225 \boxed{>}\ 181$ obtuse △

b) 9, 12, 15

✓ $9+12>15$
✓ $12+15>9$
✓ $15+9>12$
yes

$15^2 \boxed{}\ 9^2+12^2$

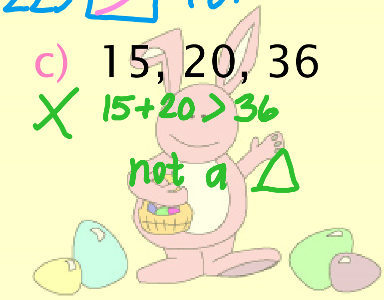
$225 \boxed{}\ 81+144$

$225 \boxed{=}\ 225$ right △

c) 15, 20, 36

✗ $15+20>36$

not a △



d) 8, 10, 12

✓ $8+10>12$
✓ $10+12>8$
✓ $12+8>10$
yes △

$12^2 \boxed{}\ 8^2+10^2$

$144 \boxed{}\ 64+100$

$144 \boxed{<}\ 164$ acute △