

8.4 Adding & Subtracting Rational Expressions

To add or subtract you need a
common denominator.

Example 1: Perform the indicated operation.

$$a) \frac{7}{4x} + \frac{3}{4x} = \frac{10 \div 2}{4x \div 2} = \frac{5}{2x}$$

$$b) \frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$$

$$c) \frac{2x^2}{x^2+1} + \frac{2}{x^2+1} = \frac{2x^2+2}{x^2+1} = \frac{2(x^2+1)}{x^2+1} = 2$$

$$d) \frac{7}{12x} - \frac{5}{12x} = \frac{2 \div 2}{12x \div 2} = \frac{1}{6x}$$

If denominators are not alike, you will need to find the least common multiple (LCM) to have a common denominator.

Example 2: Find the least common multiple of...

$$a) 4x^2 - 16 \text{ and } 6x^2 - 24x + 24$$

$$4(x^2 - 4) \quad 6(x^2 - 4x + 4)$$

$$4(x+2)(x-2) \quad 6(x-2)(x-2)$$

$$2(x-2) \cdot 2 \cdot 3 \cdot (x+2)(x-2) \rightarrow 12(x-2)(x-2)(x+2)$$

$$b) 5x^2 - 45 \text{ and } 4x^2 + 24x + 36$$

$$5(x^2 - 9) \quad 4(x^2 + 6x + 9)$$

$$5(x+3)(x-3) \quad 4(x+3)(x+3)$$

$$5 \cdot 4 \cdot (x+3)(x+3)(x-3) \rightarrow 20(x+3)(x+3)(x-3)$$

Example 3: Perform the indicated operation.

$$\frac{7(x+1)}{9x^2(x+1)} + \frac{x \cdot 3x}{(3x^2 + 3x) \cdot 3x}$$

LCD: $3x \cdot 3x(x+1) \rightarrow 9x^2(x+1)$

$$\frac{7x+7}{9x^2(x+1)} + \frac{3x^2}{9x^2(x+1)} = \boxed{\frac{3x^2+7x+7}{9x^2(x+1)}}$$

Example 4: Perform the indicated operation.

$$\frac{3(x-2)}{10x^2(x-2)} - \frac{2x \cdot 2x}{5x^2 - 10x \cdot 2x}$$

$10x^2 = 2 \cdot 5 \cdot x \cdot x$
 $5x^2 - 10x = 5x(x-2)$
 LCD = $5x \cdot 2x \cdot (x-2) = 10x^2(x-2)$

$$\frac{3x-6}{10x^2(x-2)} - \frac{-4x^2}{10x^2(x-2)} = \boxed{\frac{-4x^2+3x-6}{10x^2(x-2)}}$$

Example 5: Perform the indicated operation.

$$\frac{x \overset{x+2}{(x+2)}}{(x-2)\overset{x+2}{(x+2)}} + \frac{-8}{x^2-4}$$

LCD: $(x-2)(x+2)$

$$\frac{x^2+2x}{(x-2)(x+2)} + \frac{-8}{(x-2)(x+2)} = \frac{x^2+2x-8}{(x-2)(x+2)} = \frac{(x+4)\overset{x-2}{(x-2)}}{(x-2)(x+2)} = \boxed{\frac{x+4}{x+2}}$$

Example 7: Perform the indicated operation.

$$\frac{6x \overset{2x+5}{(2x+5)}}{(3x-1)\overset{2x+5}{(2x+5)}} - \frac{4x \overset{3x-1}{(3x-1)}}{(2x+5)\overset{3x-1}{(3x-1)}}$$

LCD: $(3x-1)(2x+5)$

$$\frac{12x^2+30x}{(3x-1)(2x+5)} + \frac{-12x^2+4x}{(3x-1)(2x+5)} = \boxed{\frac{34x}{(3x-1)(2x+5)}}$$

Example 6: Perform the indicated operation.

$$\frac{x(x-6)}{(x+6)(x-6)} + \frac{72}{x^2-36}$$

~~$\frac{x+6}{(x+6)(x-6)}$~~
LCD: $(x+6)(x-6)$

$$\frac{x^2-6x}{(x+6)(x-6)} + \frac{72}{(x+6)(x-6)} = \boxed{\frac{x^2-6x+72}{(x+6)(x-6)}}$$

Example 8: Perform the indicated operation.

$$\frac{x(x+1)}{(3x-3)(x+1)} - \frac{x \cdot 3}{(x^2-1) \cdot 3}$$

~~$\frac{3(x-1)}{(x+1)(x-1)}$~~
LCD: $3(x-1)(x+1)$

$$\frac{x^2+x}{3(x-1)(x+1)} + \frac{-3x}{3(x-1)(x+1)} = \boxed{\frac{x^2-2x}{3(x-1)(x+1)} = \frac{x(x-2)}{3(x-1)(x+1)}}$$

Example 9: Perform the indicated operation.

$$\frac{4(x+2)}{(x+1)(x+2)} - \frac{2(x+1)}{(x+2)(x+1)}$$

LCD: $(x+1)(x+2)$

$$\frac{4x+8}{(x+1)(x+2)} + \frac{-2x-2}{(x+1)(x+2)} = \boxed{\frac{2x+6}{(x+1)(x+2)}}$$

Example 10: Perform the indicated operation.

$$\frac{x \cdot 12}{(x^2 - x - 12) \cdot 12} + \frac{5(x+3)}{12(x-4)(x+3)}$$

LCD: $12(x-4)(x+3)$

$$\frac{12x}{12(x-4)(x+3)} + \frac{5x+15}{12(x-4)(x+3)} = \boxed{\frac{17x+15}{12(x-4)(x+3)}}$$

Example 11: Perform the indicated operation.

$$\frac{(x+2)^{\cancel{(x-3)}}}{(2x-2)^{\cancel{(x-3)}}} - \frac{(-2x-1) \cdot 2^{\cancel{(x-3)}}}{(x^2-4x+3) \cdot 2^{\cancel{(x-3)}}} \quad \text{LCD: } 2(x-1)(x-3)$$

$$\frac{x^2-x-6}{2(x-1)(x-3)} + \frac{+4x+2}{2(x-1)(x-3)} = \frac{x^2+3x-4}{2(x-1)(x-3)} = \frac{(x-1)(x+4)}{2(x-1)(x-3)} = \boxed{\frac{x+4}{2(x-3)}}$$

Example 12: Perform the indicated operation.

$$\frac{(x+1)^{\cancel{(x-4)}}}{(x-2)^{\cancel{(x-4)}}} + \frac{(x+2)^{\cancel{(x-2)}}}{(x-4)^{\cancel{(x-2)}}} + \frac{16-5x}{x^2-6x+8} \quad \text{LCD: } (x-4)(x-2)$$

$$\frac{x^2-3x-4}{(x-2)(x-4)} + \frac{x^2-4}{(x-4)(x-2)} + \frac{16-5x}{(x-4)(x-2)} = \frac{2(x^2-4x+4) + 2x^2-8x+8 + 16-5x}{(x-4)(x-2)}$$

$$\frac{2(x-2)(x-2)}{(x-4)(x-2)} = \boxed{\frac{2(x-2)}{x-4}}$$

Example 13: Simplify.

$$\frac{5}{x+4} \qquad \frac{5}{x+4}$$

$$\frac{1 \cdot x}{(x+4) \cdot x} + \frac{2(x+4)}{x(x+4)} \rightarrow \frac{x}{x(x+4)} + \frac{2x+8}{x(x+4)} \rightarrow \frac{3x+8}{x(x+4)}$$

$$\frac{5}{x+4} \cdot \frac{x(x+4)}{3x+8} = \boxed{\frac{5x}{3x+8}}$$

$(x+4)x \cdot \cancel{3x+8}$

Example 14: Simplify.

$$\frac{3}{x+5} \qquad \frac{3}{x+5}$$

$$\frac{2(x+5)}{(x-3)(x+5)} + \frac{1(x-3)}{(x+5)(x-3)} \rightarrow \frac{2x+10}{(x+5)(x-3)} + \frac{x-3}{(x+5)(x-3)} = \frac{3x+7}{(x+5)(x-3)}$$

$$\frac{3}{x+5} \cdot \frac{(x+5)(x-3)}{3x+7} = \boxed{\frac{3(x-3)}{3x+7}}$$