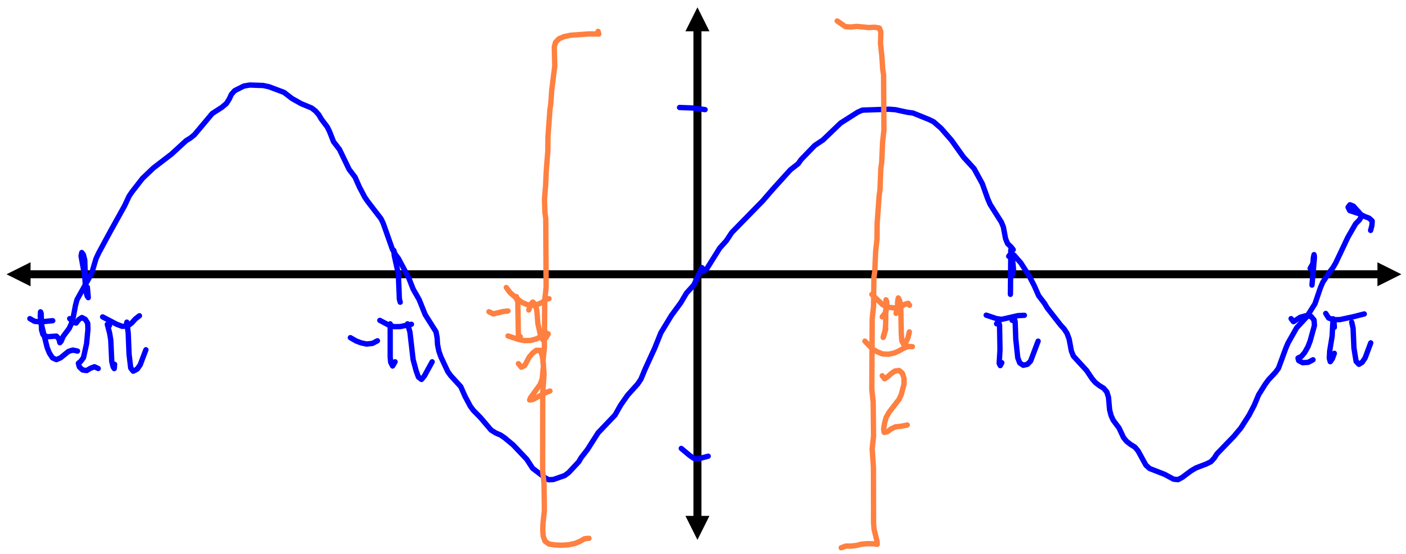
5.5 Inverse Trigonometric Functions

REMEMBER: An inverse reverses (undoes) a function.

Inverse Sine Function

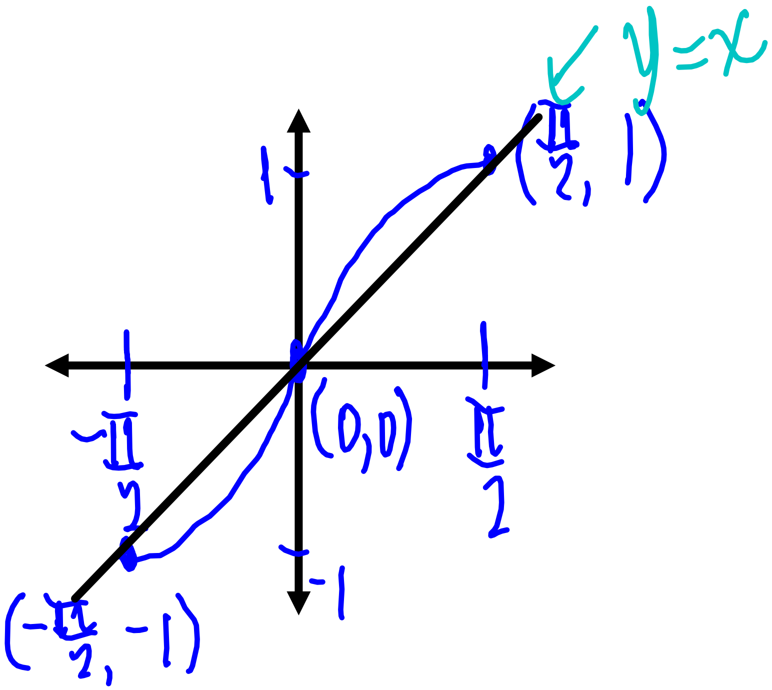
First, graph y = sin x.



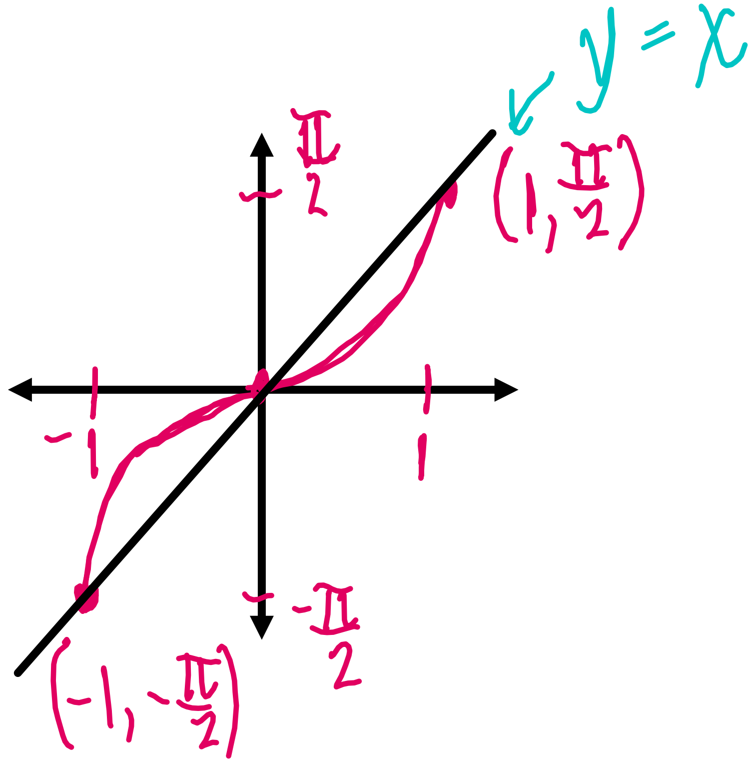
A function must pass the ___HLT___ in order for the inverse to be a function.

So we will restrict the domain of the sine graph to $\underline{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$.

Now, graph $y = \sin x$ under these restrictions.



Then, graph $y = \sin^{-1}x$.



Graph $y = \sin^{-1}x$ by reflecting the graph of $y = \sin x$ in the line $\underline{y=x}$.

Definition of the Inverse Sine Function

The inverse sine function is the function $\sin^{-1}$ with domain $[-1,1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$\sin^{-1} x$

The inverse sine function is also called arcsine and is denoted arcsin.

Evaluating the Inverse Sine Function

All answers must be in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Find $\sin^{-1}(\frac{1}{2})$.

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}}$

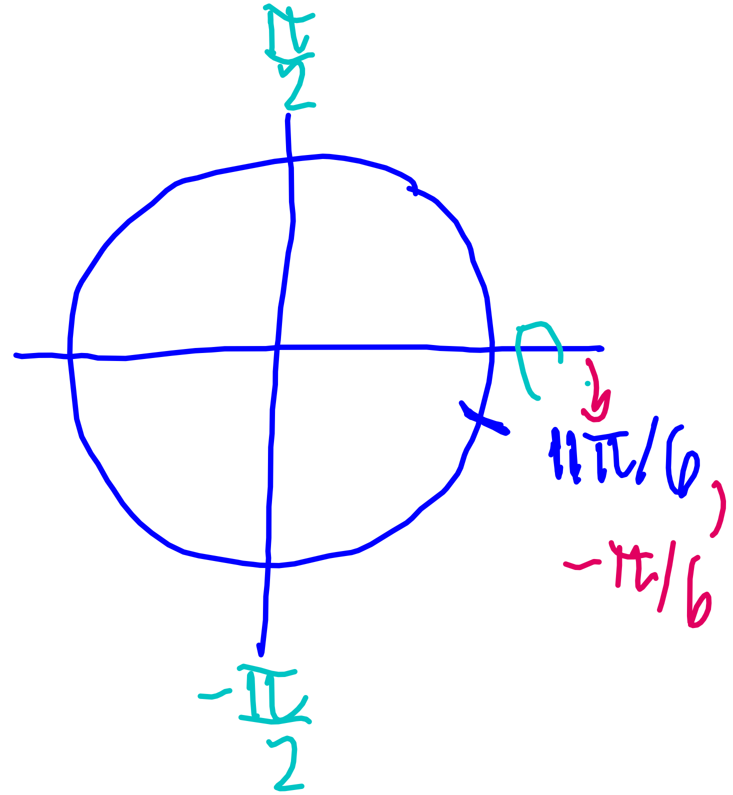
y-value of orig. function

Find $\sin^{-1}(-\frac{1}{2})$.

$\sin x = -\frac{1}{2}$

$\cancel{x = \frac{11\pi}{6}}$

$\boxed{x = -\frac{\pi}{6}}$



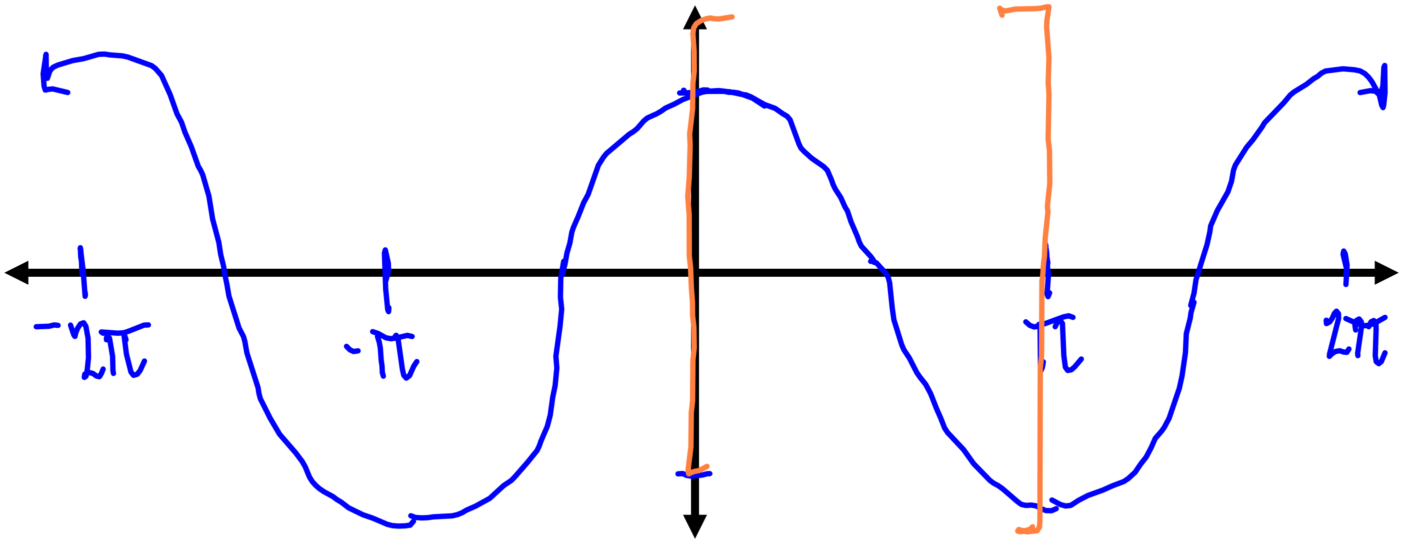
Use a calculator to find approximate values.

$\sin^{-1}(.82) \approx 0.961$

$\sin^{-1}\left(\frac{1}{3}\right) \approx 0.340$

Inverse Cosine Function

First, graph y = cos x.

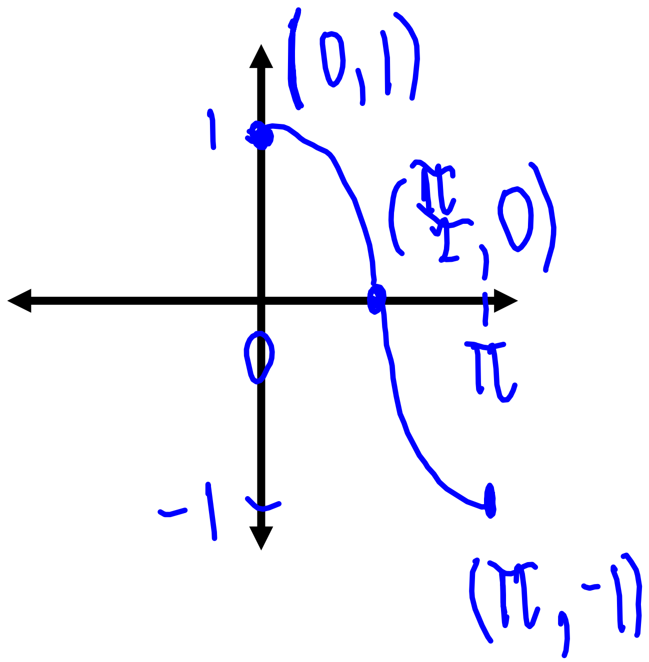


Just like y = sin x, y = cos x is not one-to-one, so we must restrict the domain.

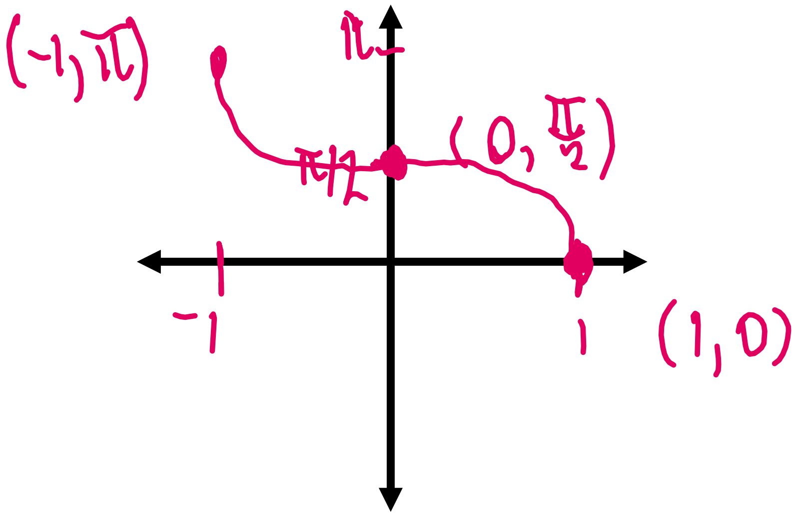
Where should we choose to restrict the domain?

So we will restrict the domain of the cosine graph to $[0, \pi]$.

Now, graph $y = \cos x$ under these restrictions.



Then, graph $y = \cos^{-1} x$.



Definition of the Inverse Cosine Function

The inverse cosine function is the function $\cos^{-1}$ with domain $[-1,1]$ and range $[0,\pi]$.

The inverse sine function is also called arccosine and is denoted arccos.

Evaluating the Inverse Cosine Function

$[0, \pi]$

Find $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. $\longrightarrow$ $\cos x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}}$

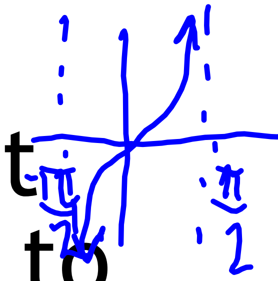
Find $\cos^{-1} 0$. $\longrightarrow$ $\cos x = 0$

$\boxed{x = \frac{\pi}{2}}$

Find $\cos^{-1}\left(\frac{5}{7}\right)$. $\longrightarrow$ $\cos x = \frac{5}{7}$ $\boxed{x \approx 0.775}$

Definition of the Inverse Tangent Function

We restrict the domain of the tangent function to the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ in order to obtain a one-to-one function.



The inverse tangent function is the function $\tan^{-1}$ with domain $\mathbb{R}$ and range $(-\frac{\pi}{2}, \frac{\pi}{2})$.

The inverse tangent function is also called arctangent and is denoted arctan.

Evaluating the Inverse Tangent Function

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Find $\tan^{-1} 1$. $\longrightarrow$ $\tan x = 1$

$\boxed{x = \frac{\pi}{4}}$

Find $\tan^{-1}\sqrt{3}$. $\longrightarrow$ $\tan x = \sqrt{3}$

$\boxed{x = \frac{\pi}{3}}$

Find $\tan^{-1}(-20)$. $\longrightarrow$ $\tan x = -20$ $\boxed{x \approx -1.521}$

Find the exact value of the expression, if it is defined.

domain
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}}{2}}$$

$\sin x = \frac{\sqrt{2}}{2}$ $\quad x = \frac{\pi}{4}$

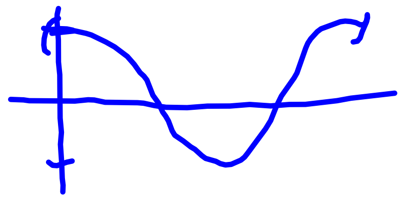
$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\cos^{-1}\frac{3}{2}\right)$$

$$\cos\left(\cos^{-1}\frac{2}{3}\right) = \boxed{\frac{2}{3}}$$

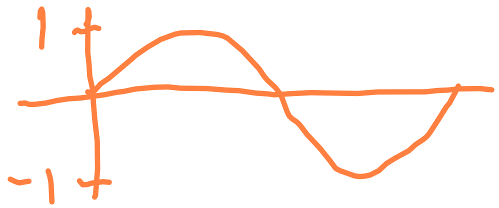
$\cos x = \frac{2}{3}$

$\cos x = \frac{3}{2}$



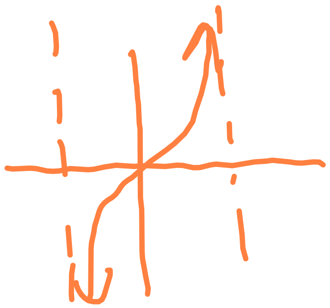
Find the exact value of the expression, if it is defined.

$\cancel{\sin(\sin^{-1}} \frac{1}{4}) = \frac{1}{4}$

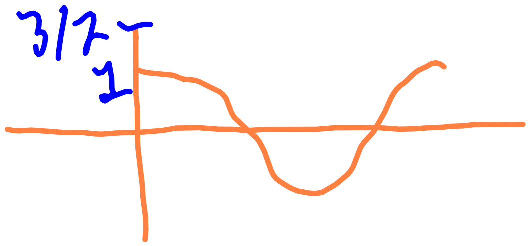


$\sin x = \frac{1}{4}$

$\cancel{\tan(\tan^{-1}} 5) = 5$



$\cos(\cos^{-1} \frac{3}{2})$ undefined



Find the exact value of the expression, if it is defined.

domain $[0, \pi]$

$$\cos^{-1}\left(\underbrace{\cos\frac{5\pi}{4}}\right)$$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \longrightarrow \cos x = -\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{3\pi}{4}}$

domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin^{-1}\left(\underbrace{\sin\left(-\frac{\pi}{6}\right)}\right)$$

$\sin^{-1}\left(-\frac{1}{2}\right) \longrightarrow \sin x = -\frac{1}{2}$

$\boxed{x = -\frac{\pi}{6}}$

domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan^{-1}\left(\underbrace{\tan\frac{2\pi}{3}}\right) \longrightarrow \tan x = -\sqrt{3}$$

$\boxed{x = -\frac{\pi}{3}}$

$\tan^{-1}\left(-\sqrt{3}\right)$

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Find the exact value of the expression, if it is defined.

$\sin^{-1}(\sin \frac{5\pi}{6})$

$\cos(\sin^{-1} \frac{\sqrt{3}}{2})$

$\tan(\sin^{-1} \frac{1}{2})$

$\sin(\tan^{-1}(-1))$