

## 6.4 Using the Elimination Method with Multiplication

### Example 1

$$\begin{array}{l}
 \downarrow \\
 -2 \cdot (2x + y) = (-9) \cdot -2 \longrightarrow -4x - 2y = 18 \\
 4x + 11y = 9 \\
 \hline
 4x + 11(3) = 9 \\
 4x + 33 = 9 \\
 \quad -33 \quad -33 \\
 \hline
 4x = -24 \\
 \frac{4x}{4} = \frac{-24}{4} \\
 x = -6
 \end{array}$$

$$\begin{array}{r}
 -4x - 2y = 18 \\
 4x + 11y = 9 \\
 \hline
 9y = 27 \\
 \frac{9y}{9} = \frac{27}{9} \\
 y = 3
 \end{array}$$

(-6, 3)

### Example 2

$$\begin{array}{l}
 \downarrow \\
 2 \cdot (3x + 5y) = (11) \cdot 2 \longrightarrow 6x + 10y = 22 \\
 -3 \cdot (2x + 3y) = (7) \cdot -3 \longrightarrow -6x - 9y = -21 \\
 \hline
 y = 1
 \end{array}$$

(2, 1)

$$\begin{array}{r}
 6x + 10(1) = 22 \\
 6x + 10 = 22 \\
 \quad -10 \quad -10 \\
 \hline
 6x = 12 \\
 \frac{6x}{6} = \frac{12}{6} \\
 x = 2
 \end{array}$$

### Example 3

$$5 \cdot (2x - 3y) = (8) \cdot 5 \longrightarrow 10x - 15y = 40$$

$$2 \cdot (-5x + 2y) = (13) \cdot 2 \longrightarrow -10x + 4y = 26$$

$$-5x + 2(-6) = 13$$

$$\begin{array}{r} -5x - 12 = 13 \\ + 12 \quad + 12 \\ \hline \end{array}$$

$$\begin{array}{r} -5x = 25 \\ \hline -5 \quad -5 \end{array}$$

$$x = -5$$

$$\begin{array}{r} -11y = 66 \\ \hline -11 \quad -11 \\ \hline y = -6 \end{array}$$

$$(-5, -6)$$

### Example 4

$$5 \cdot (3x - 7y) = (5) \cdot 5 \longrightarrow 15x - 35y = 25$$

$$-5x = -9y + 5$$

$$\begin{array}{r} +9y \quad +9y \\ \hline \end{array}$$

$$3 \cdot (-5x + 9y) = (5) \cdot 3 \longrightarrow -15x + 27y = 15$$

$$15x - 35(-5) = 25$$

$$\begin{array}{r} 15x + 175 = 25 \\ - 175 \quad - 175 \\ \hline \end{array}$$

$$\begin{array}{r} 15x = -150 \\ \hline 15 \quad 15 \end{array}$$

$$x = -10$$

$$\begin{array}{r} -8y = 40 \\ \hline -8 \quad -8 \end{array}$$

$$y = -5$$

$$(-10, -5)$$

**Example 5**

$$\begin{array}{l}
 4 \cdot (2x - 3y) = (6) \cdot 4 \longrightarrow 8x - 12y = 24 \\
 \quad 4y = -7x - 8 \\
 \quad + 7x \quad + 7x \\
 3 \cdot (7x + 4y) = (-8) \cdot 3 \longrightarrow \frac{21x + 12y = -24}{\hline} \\
 \quad \frac{29x}{29} = \frac{0}{29} \\
 \quad \quad \quad x = 0 \\
 \\
 \begin{array}{r}
 \cancel{8(0)} - 12y = 24 \\
 -12y = 24 \\
 \hline
 -12 \quad -12 \\
 y = -2
 \end{array} \\
 \\
 \quad \quad \quad (0, -2)
 \end{array}$$

**Example 6**

$x$  = price per pack of hot dogs  
 $y$  = price per lb of ground beef

Two women are shopping for items for their separate cookouts. The first woman purchases 4 packs of hot dogs and 5 lbs of ground beef for hamburgers, and she pays a total of \$33. The second woman purchases 3 packs of hot dogs and 7 lbs of ground beef, and she pays a total of \$41. Find out how much each pack of hot dogs cost and how much the ground beef is per pound.

$$\begin{array}{l}
 -3 \cdot (4x + 5y) = (33) \cdot -3 \longrightarrow -12x - 15y = -99 \\
 4 \cdot (3x + 7y) = (41) \cdot 4 \longrightarrow \frac{12x + 28y = 164}{\hline} \\
 \\
 \begin{array}{r}
 12x + 28(5) = 164 \\
 12x + 140 = 164 \\
 \hline
 -140 \quad -140 \\
 \hline
 12x = 24 \\
 \frac{12}{12} = \frac{24}{12} \\
 x = 2
 \end{array} \\
 \\
 \begin{array}{r}
 \frac{13y}{13} = \frac{65}{13} \\
 y = 5
 \end{array} \\
 \\
 \text{\$2/pack of hot dogs} \\
 \text{\$5/lb of ground beef}
 \end{array}$$

### Example 7

$x =$  cost for 1-night stay  
 $y =$  cost per meal

A Beach Resort is offering two weekend specials. One includes a 2-night stay with 3 meals and costs \$195. The other includes a 3-night stay with 5 meals and costs \$300. What is the cost of a 1-night stay? What is the cost per meal?

$$\begin{array}{r} 5 \cdot (2x + 3y) = (195) \cdot 5 \longrightarrow 10x + 15y = 975 \\ -3 \cdot (3x + 5y) = (300) \cdot -3 \longrightarrow -9x - 15y = -900 \\ \hline x = 75 \end{array}$$

$$\begin{array}{r} 10(75) + 15y = 975 \\ 750 + 15y = 975 \\ -750 \phantom{+ 15y} = -750 \\ \hline 15y = 225 \\ \underline{15} \quad \underline{15} \\ y = 15 \end{array}$$

\$75/night  
 \$15/meal

### Example 8

$x =$  # of students/van  
 $y =$  # of students/bus

The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

$$\begin{array}{r} 8x + 8y = 240 \longrightarrow 8x + 8y = 240 \\ -8 \cdot (4x + y) = (54) \cdot -8 \longrightarrow -32x - 8y = -432 \\ \hline -24x \phantom{+ 8y} = -192 \\ \underline{-24} \phantom{+ 8y} \quad \underline{-24} \end{array}$$

$$\begin{array}{r} 4(8) + y = 54 \\ 32 + y = 54 \\ -32 \phantom{+ y} = -32 \\ \hline y = 22 \end{array}$$

8 students/van  
 22 students/bus

$$x = 8$$