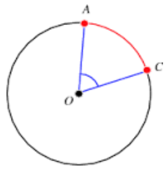
6.1 Part 2

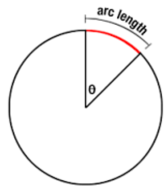
Arc - a piece of a circle

An arc subtends (is opposite from) the central angle.



In a circle of radius r, the length s of an arc that subtends a central angle of θ is:

$$\boxed{\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360 \text{ or } 2\pi}}$$



To find circumference of a circle, use $C = 2\pi r$.

Example 1

Find the length of an arc of a circle with radius 10 meters that subtends a central angle of 30°.

s

$C = 2\pi r$
$C = 2\pi(10)$
$C = 20\pi$

$$\frac{s}{20\pi} = \frac{30}{360}$$

$$\frac{360s}{360} = \frac{60\cancel{0}\pi}{36\cancel{0}}$$

$$\boxed{s = \frac{5\pi}{3} m}$$

Example 2

A central angle θ in a circle with radius 4 meters is subtended by an arc of length 6 meters. Find the measure of θ in radians.

s

$C = 2\pi r$
$C = 2\pi(4)$
$C = 8\pi$

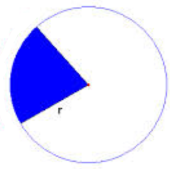
$$\frac{6}{8\pi} = \frac{\theta}{2\pi}$$

$$\frac{8\pi\theta}{8\pi} = \frac{12\cancel{\pi}}{8\cancel{\pi}}$$

$$\boxed{\theta = \frac{3}{2} \text{ rad}}$$

Area of a Circular Sector

$$\boxed{\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{360 \text{ or } 2\pi}}$$



Sector Area

To find area of a circle, use $\underline{A = \pi r^2}$.

Example 3

Find the area of a sector of a circle with central angle 60° if the radius is 3 meters.

$A = \pi r^2$

$A = \pi \cdot 3^2$

$A = 9\pi$

$$\frac{A}{9\pi} = \frac{60}{360}$$

$$\frac{360A}{360} = \frac{54\cancel{0}\pi}{36\cancel{0}}$$

$$\boxed{A = \frac{3\pi}{2} m^2}$$

Suppose a point moves along a circle.
There are 2 ways to describe the motion of the point:
linear speed and angular speed.

Linear speed: the rate at which the distance traveled is changing, so linear speed is the distance traveled divided by the time elapsed.

Linear speed: $v = \frac{s}{t}$ $\frac{\text{linear dist.}}{\text{time}}$

Angular speed: the rate at which the central angle is changing, so angular speed is the number of radians this angle changes divided by the time elapsed.

Angular speed: $\omega = \frac{\theta}{t}$ $\frac{\text{angle}}{\text{time}}$

Example 4

A boy rotates a stone in a 3-foot-long (radius) sling at a rate of 15 revolutions every 10 seconds. Find the angular and linear velocities of the stone.

linear speed

1 revolution $= 2\pi r$ ($r = 3$)

1 rev $= 6\pi$

15 rev $= 15 \cdot 6\pi = 90\pi \leftarrow s$ (linear dist)

$$v = \frac{s}{t} = \frac{90\pi}{10\,\text{sec}} = \boxed{9\pi\,\text{ft/sec} = v}$$

angular speed

1 rev $= 2\pi$

15 rev $= 15 \cdot 2\pi = 30\pi$

$$\omega = \frac{\theta}{t} = \frac{30\pi}{10\,\text{sec}} = \boxed{3\pi\,\text{rad/sec} = \omega}$$

Can we write a formula that allows us to find linear speed based on the angular speed?

Use $\boxed{\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360 \text{ or } 2\pi}}$ to solve for s.

$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

$$\frac{2\pi s}{2\pi} = \frac{\cancel{2\pi} r\theta}{\cancel{2\pi}}$$

$$s = r\theta$$

$v = \frac{s}{t}$ AND $s = r\theta$

therefore $v = \frac{r\theta}{t}$ AND $\omega = \frac{\theta}{t}$

Thus, we have the formula $v = r\omega$.

Example 5

$r = 13$ in

A woman is riding a bicycle whose wheels are 26 inches in diameter. If the wheels rotate at 125 revolutions per minute (rpm), find the speed at which she is traveling in mi/h. ← linear speed

1 rev $= 2\pi \cdot 13 = 26\pi$

125 rev $= 3250\pi$

$$v = \frac{s}{t} = \frac{3250\pi \text{ in}}{1 \text{ min}}$$

$$\frac{3250\pi \cancel{\text{in}}}{\cancel{\text{min}}} \cdot \frac{1 \cancel{\text{ft}}}{12 \cancel{\text{in}}} \cdot \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} = \frac{195{,}000\pi \text{ mile}}{63360 \text{ hr}} \approx$$

$$\boxed{9.7 \text{ miles/hr}}$$