

8.3 Multiply & Divide Rational Expressions

A rational expression is in simplified form if its numerator and denominator have no common factors (other than ± 1).

Example $\frac{15}{20} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{3}{4}$

Example $\frac{ac}{bc} = \frac{a}{b}$

Example 1

Simplify: $\frac{2(\cancel{x+1})}{(\cancel{x+1})(x+3)} = \frac{2}{x+3}$

Example 2

Simplify: $\frac{40x + 20}{10x + 30}$

$\frac{20 \overset{\div 10}{(2x+1)}}{10 \overset{\div 10}{(x+3)}} = \frac{2(2x+1)}{x+3}$

Example 3

$$\text{Simplify: } \frac{x^2 - 2x - 15}{x^2 - 9} = \frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$$

$$\boxed{\frac{x-5}{x-3}}$$

Example 4

$$\text{Simplify: } \frac{x^2 - 2x - 3}{x^2 - x - 6} = \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x+2)}$$

$$\boxed{\frac{x+1}{x+2}}$$

Multiplying Rational Expressions

The rule is the *same* as for multiplying fractions.

Example 5

$$\text{Multiply: } \frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$$

$$\frac{56x^7y^4}{8xy^3} \quad \boxed{7x^6y}$$

Example 6

$$\text{Multiply: } \frac{15x^2y^3}{3xy^4} \cdot \frac{27x^5}{35x^4y}$$

$$\frac{9x^7y^3}{3x^5y^5} \quad 3x^2y^{-2} \rightarrow \boxed{\frac{3x^2}{y^2}}$$

Example 7

Multiply: $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$

$$\frac{\cancel{3x} \cancel{(1-x)}}{\cancel{(x+5)} \cancel{(x-1)}} \cdot \frac{\cancel{(x+5)} \cancel{(x-4)}}{\cancel{3x}}$$

$$-(x-4)$$

$$\boxed{-x+4}$$

Example 8

Multiply: $\frac{20x - 5x^2}{x^2 - x} \cdot \frac{x^2 + 3x - 4}{x^2 - 16}$

$$\frac{\cancel{5x} \cancel{(4-x)}}{\cancel{x} \cancel{(x-1)}} \cdot \frac{\cancel{(x+4)} \cancel{(x-1)}}{\cancel{(x+4)} \cancel{(x-4)}}$$

$$\boxed{-5}$$

Example 9 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Multiply: $\frac{x+2}{x^3 - 27} \cdot (x^2 + 3x + 9)$

$$\frac{x+2}{(x-3)(x^2+3x+9)} \cdot \frac{x^2+3x+9}{1}$$

$$\boxed{\frac{x+2}{x-3}}$$

Dividing Rational Expressions

Same as dividing fractions...*multiply by the reciprocal.*

Example 10

Divide: $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$

$$\frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x}$$

$$\frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)}$$

$$\boxed{\frac{7}{2}}$$

Example 11

Divide:

$$\frac{4x}{5x-20} \div \frac{x^2-2x}{x^2-6x+8}$$

$$\frac{4x}{5x-20} \cdot \frac{x^2-6x+8}{x^2-2x}$$

$$\frac{4x}{5(x-4)} \cdot \frac{(x-4)(x-2)}{x(x-2)}$$

$$\boxed{\frac{4}{5}}$$

Example 12

Divide:

$$\frac{6x^2+x-15}{4x^2} \div \frac{(3x^2+5x)}{3x^2+5x}$$

$$\frac{6x^2+x-15}{4x^2} \cdot \frac{1}{3x^2+5x}$$

$$\frac{(2x-3)(3x+5)}{4x^2} \cdot \frac{1}{x(3x+5)}$$

$$\boxed{\frac{2x-3}{4x^3}}$$

$$\frac{-9}{6} \downarrow \frac{10}{6}$$

$$\frac{-3}{2} \downarrow \frac{5}{3}$$

A *complex fraction* is a quotient that contains one or more fractions in the numerator, the denominator, or both.

Example 13

Simplify: $\frac{4a^2 - 1}{a^2 - 4} \cdot \frac{2a - 1}{a + 2}$

$$\frac{4a^2 - 1}{a^2 - 4} \cdot \frac{2a - 1}{a + 2}$$

$$\frac{(2a-1)(2a+1)}{(a-2)(a+2)} \cdot \frac{2a-1}{a+2}$$

$$\frac{2a+1}{a-2}$$

Example 14

Simplify: $\frac{x^2 + 10x - 11}{x^2 + 6x + 5} \cdot \frac{x^2 + 9x - 22}{x^2 + 3x - 10}$

$$\frac{x^2 + 10x - 11}{x^2 + 6x + 5} \cdot \frac{x^2 + 9x - 22}{x^2 + 3x - 10}$$

$$\frac{(x+11)(x-1)}{(x+5)(x+1)} \cdot \frac{(x+5)(x-2)}{(x+1)(x-2)}$$

$$\frac{x-1}{x+1}$$