6.1 Part 1: Angle Measure

The measure of an angle is how much the angle "opens." This can be measured in degrees or radians (abbreviated rad).

1 rad ≈ 57.296 degrees

To convert degrees to radians, multiply by $\frac{\pi}{180}$

To convert radians to degrees, multiply by $\frac{180}{\pi}$

Example 1

a) Express 60° in radians.

$60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$

b) Express $\frac{\pi}{6}$ in degrees.

$\frac{\pi}{6} \cdot \frac{180}{\pi} = 30°$

Example 2

Find the radian measure of the angle with the given degree measure.

a) 36°

$36 \cdot \frac{\pi}{180} = \frac{\pi}{5}$

b) -480°

$-480 \cdot \frac{\pi}{180} = -\frac{8\pi}{3}$

c) 60°

d) -135°

$-135 \cdot \frac{\pi}{180} = -\frac{3\pi}{4}$

Example 3

Find the degree measure of the angle with the given radian measure.

a) $\frac{3\pi}{4} \cdot \frac{180}{\pi}$

$135°$

b) $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150°$

c) $-1.5 \cdot \frac{180}{\pi}$

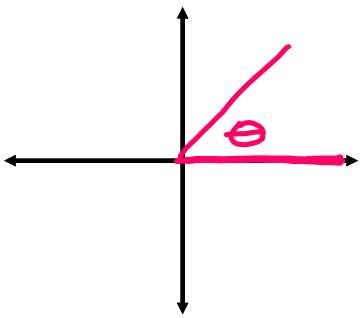
$-\frac{3}{2} \cdot \frac{180}{\pi}$

$-\frac{270}{\pi}$

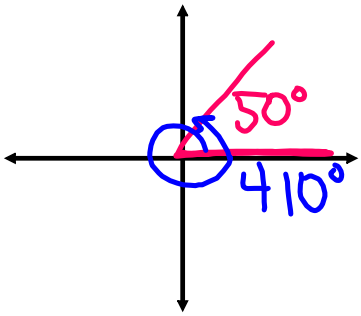
d) $-\frac{\pi}{12} \cdot \frac{180}{\pi}$

$-15°$

An angle is in standard position if it is drawn with its vertex at the origin and initial side on the positive x-axis.



Two angles in standard position are coterminal if their sides coincide (end at same line).



To find positive angles that are coterminal with an angle, we add any multiple of 360°.

Find angles that are coterminal with angle $\theta = 30°$ in standard position.

$30°, 390°, 750°, 1110°, \ldots$

Find angles that are coterminal with angle $\theta = \frac{\pi}{3}$ in standard position.

$\frac{\pi}{3} + 2\pi = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3} + \frac{6\pi}{3} = \frac{13\pi}{3} + \frac{6\pi}{3}$

$\frac{7\pi}{3}, \frac{13\pi}{3}, \frac{19\pi}{3}, \ldots$ How do we find negative angles?

subtract

Example 4

Find an angle with measure between 0° and 360° that is coterminal with the angle measure 1290° in standard position.

start $1290°$ $930°$ $570°$

-360 -360 -360

$930°$ $570°$ $210°$ end

Example 5

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal.

a) 135° ± 360

$-585^\circ, -225^\circ, 135^\circ, 495^\circ, 855^\circ$

b) $\frac{3\pi}{4}$ $\pm 2\pi \longrightarrow \pm \frac{8\pi}{4}$

$-\frac{13\pi}{4}, -\frac{5\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}$

c) $-\frac{\pi}{4}$ $\pm 2\pi \longrightarrow \pm \frac{8\pi}{4}$

$-\frac{17\pi}{4}, -\frac{9\pi}{4}, -\frac{\pi}{4}, \frac{7\pi}{4}, \frac{15\pi}{4}$

Example 6

The measure of two angles in standard position are given. Determine whether the angles are coterminal.

a) $-30^\circ, 330^\circ$ coterminal

$-30 + 360 \stackrel{?}{=} 330$ ✓

b) $\frac{32\pi}{3}, \frac{11\pi}{3}$ not coterminal

$\frac{32\pi}{3} - \frac{6\pi}{3} = \frac{26\pi}{3} - \frac{6\pi}{3} = \frac{20\pi}{3} - \frac{6\pi}{3} = \frac{14\pi}{3} - \frac{6\pi}{3} = \frac{8\pi}{3}$

c) $50^\circ, 340^\circ$ not coterminal

$50 + 360 \neq 340$

Example 7

Find an angle between 0° and 360° that is coterminal with the given angle.

a) 361°

$361 - 360 = \boxed{1^\circ}$

b) -100°

$-100 + 360 = \boxed{260^\circ}$

c) 1270°

$1270 - 360 = 910 - 360 = 550 - 360 = \boxed{190^\circ}$

Example 8

Find an angle between 0 and 2π that is coterminal with the given angle. $\pm 2\pi$

a) $-\frac{7\pi}{3} + \frac{6\pi}{3} = -\frac{\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{5\pi}{3}}$

b) $\boxed{10 - 2\pi}$

c) $\frac{51\pi}{2} - \frac{4\pi}{2} = \frac{47\pi}{2} - \frac{4\pi}{2} = \frac{43\pi}{2} - \frac{4\pi}{2} = \frac{39\pi}{2} - \frac{4\pi}{2} =$

$\frac{35\pi}{2} - \frac{4\pi}{2} = \frac{31\pi}{2} - \frac{4\pi}{2} = \frac{27\pi}{2} - \frac{4\pi}{2} = \frac{23\pi}{2} - \frac{4\pi}{2} =$

$\frac{19\pi}{2} - \frac{4\pi}{2} = \frac{15\pi}{2} - \frac{4\pi}{2} = \frac{11\pi}{2} - \frac{4\pi}{2} = \frac{7\pi}{2} - \frac{4\pi}{2} = \boxed{\frac{3\pi}{2}}$