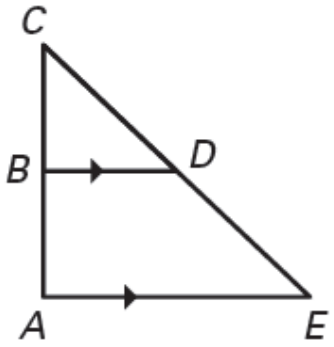
6.5 Use Proportionality Theorems

Theorem 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



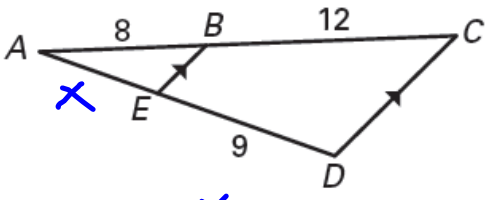
IF $\overline{BD} \parallel \overline{AE}$

THEN $\frac{CB}{BA} = \frac{CD}{DE}$

$\frac{CB}{CD} = \frac{BA}{DE}$

Example 1

What is the length of $\overline{AE}$?



$\frac{8}{12} = \frac{x}{9}$

$\frac{8}{x} \times \frac{12}{9}$

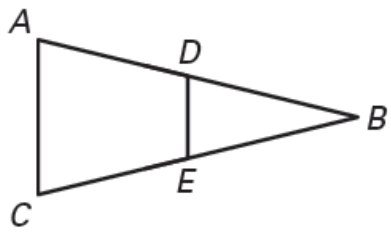
$\frac{12x}{12} = \frac{72}{12}$

$x = 6$

$AE = 6$

Theorem 6.5 Triangle Proportionality Converse

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

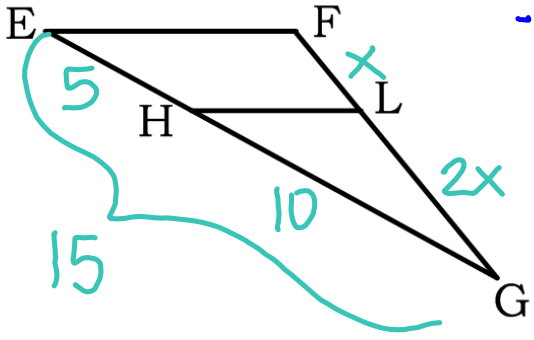


IF $\frac{AD}{CE} = \frac{DB}{EB}$ or $\frac{AD}{DB} = \frac{CE}{EB}$

THEN $\overline{AC} \parallel \overline{DE}$

Example 2

In Triangle *EFG*, *EG* = 15, *EH* = 5, and *LG* is twice *FL*. Determine whether $\overline{HL} \parallel \overline{EF}$.



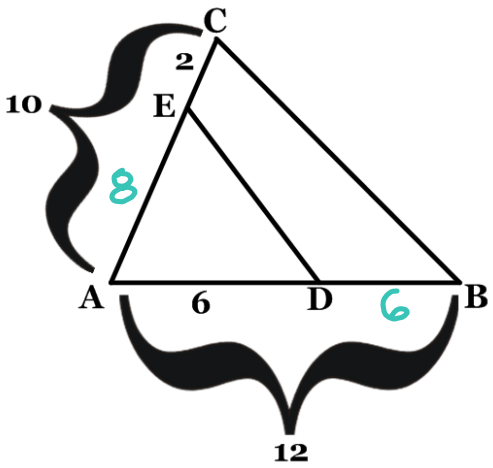
$\frac{5}{x} \stackrel{?}{=} \frac{10}{2x}$

$10x = 10x$ ✓

$\overline{HL} \parallel \overline{EF}$

Example 3

In the figure below, $CA = 10$, $CE = 2$, $DA = 6$, and $BA = 12$. Determine if $\overline{ED} \parallel \overline{CB}$.



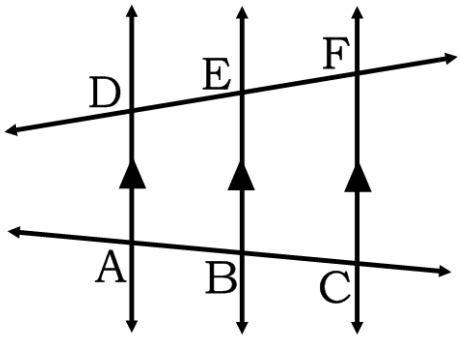
$\frac{8}{6} \stackrel{?}{=} \frac{2}{6}$

$48 \neq 12$

$\overline{ED}$ not $\parallel$ $\overline{CB}$

Theorem 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



IF $\overline{DA} \parallel \overline{EB} \parallel \overline{FC}$

THEN $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$

or

$\frac{DE}{EF} = \frac{AB}{BC}$

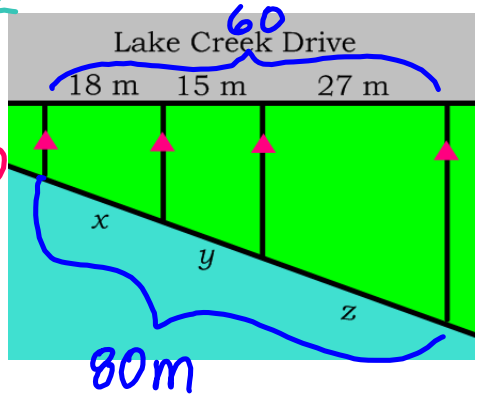
Example 4

In Lake Creek, the lots on which houses are to be built are laid out as shown. What is the lake frontage for each of the three lots if the total frontage is 80 meters?

$\frac{18}{x} = \frac{3}{4}$ $3x = 72$ $x = 24m$

$\frac{15}{y} = \frac{3}{4}$ $3y = 60$ $y = 20m$

$\frac{27}{z} = \frac{3}{4}$ $3z = 108$ $z = 36m$

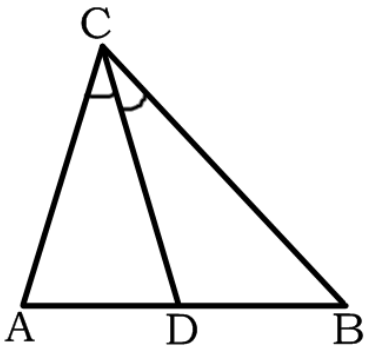


$\frac{18}{x}$ $\frac{15}{y}$ $\frac{27}{z}$

$\frac{60}{80} \downarrow \frac{3}{4}$

Theorem 6.7 Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



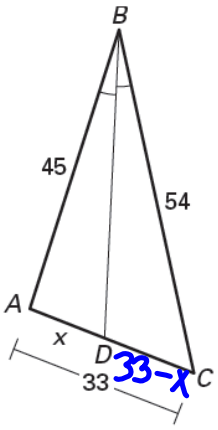
IF $\overline{CD}$ is $\angle$ bisector

THEN $\frac{CA}{AD} = \frac{CB}{DB}$

$\frac{CA}{CB} = \frac{AD}{DB}$

Example 5

Find the length of $\overline{AD}$.



$\frac{45}{x} = \frac{54}{33-x}$

$\frac{45 \div 9}{54 \div 9} = \frac{x}{33-x}$

$\frac{5}{6} = \frac{x}{33-x}$

$5(33-x) = 6x$

$165 - 5x = 6x$

$+5x \quad +5x$

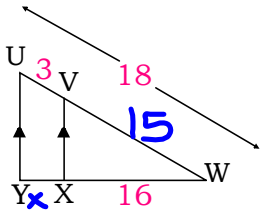
$\frac{165}{11} = \frac{11x}{11}$

$15 = x$

AD = 15

Practice

1. What is the length of $\overline{YX}$?



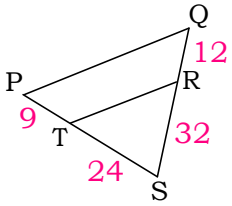
$\frac{3}{x} = \frac{15}{16}$

$\frac{15x}{15} = \frac{48}{15}$

$x = \frac{16}{5}$

$YX = \frac{16}{5}$

2. Determine whether $\overline{PQ} \parallel \overline{TR}$.

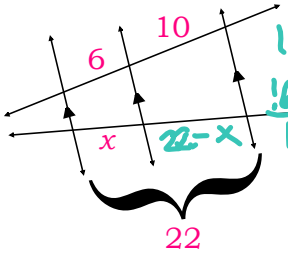


$\frac{9}{24} \stackrel{?}{=} \frac{12}{32}$ ✓

$\frac{9}{12} \stackrel{?}{=} \frac{24}{32}$ ✓

$\overline{PQ} \parallel \overline{TR}$

3. What is x?



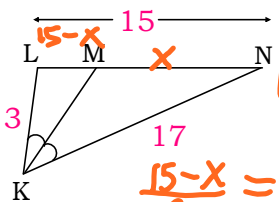
$\frac{6}{x} = \frac{10}{22-x}$

$10x = 132 - 6x$

$\frac{16x}{16} = \frac{132}{16}$

$x = \frac{33}{4}$

4. Find the length of $\overline{MN}$.



$MN = \frac{51}{4}$

$\frac{15-x}{3} = \frac{x}{17}$

$255 - 17x = 3x$

$\frac{51}{4} = \frac{255}{20} - \frac{20x}{20}$