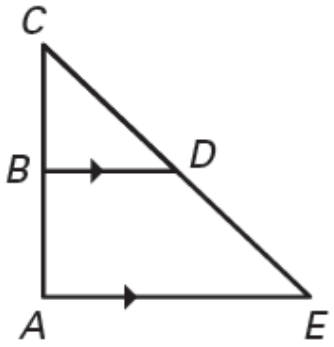
6.5 Use Proportionality Theorems

Theorem 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



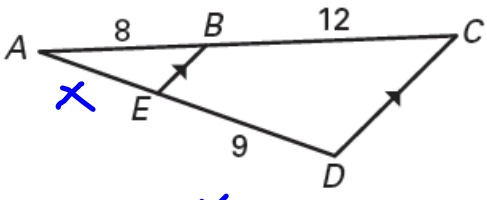
IF $\overline{BD} \parallel \overline{AE}$

THEN $\frac{CB}{BA} = \frac{CD}{DE}$

$\frac{CB}{CD} = \frac{BA}{DE}$

Example 1

What is the length of $\overline{AE}$?



$\frac{8}{12} = \frac{x}{9}$

$\frac{8}{x} \times \frac{12}{9}$

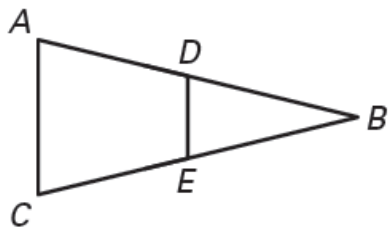
$\frac{12x}{12} = \frac{72}{12}$

$x = 6$

$AE = 6$

Theorem 6.5 Triangle Proportionality Converse

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

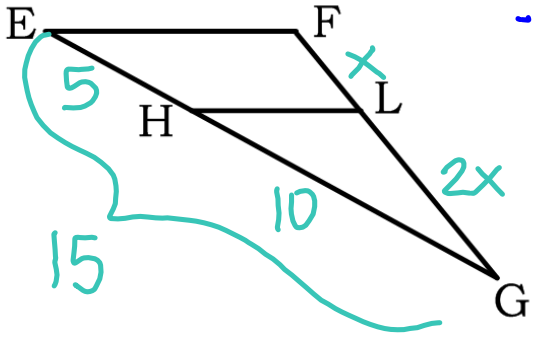


IF $\frac{AD}{CE} = \frac{DB}{EB}$ or $\frac{AD}{DB} = \frac{CE}{EB}$

THEN $\overline{AC} \parallel \overline{DE}$

Example 2

In Triangle *EFG*, *EG* = 15, *EH* = 5, and *LG* is twice *FL*. Determine whether $\overline{HL} \parallel \overline{EF}$.



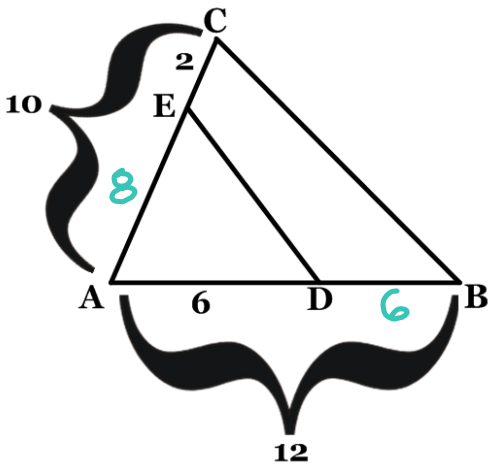
$\frac{5}{x} \stackrel{?}{=} \frac{10}{2x}$

$10x = 10x$ ✓

$\overline{HL} \parallel \overline{EF}$

Example 3

In the figure below, $CA = 10$, $CE = 2$, $DA = 6$, and $BA = 12$. Determine if $\overline{ED} \parallel \overline{CB}$.



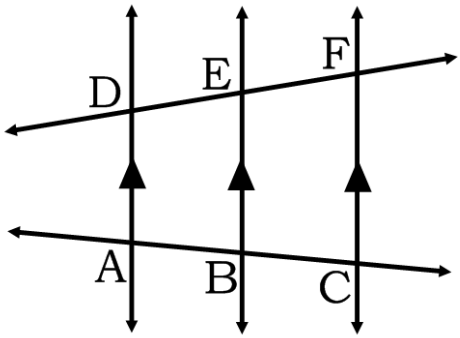
$\frac{8}{6} \stackrel{?}{=} \frac{2}{6}$

$48 \neq 12$

$\overline{ED}$ not $\parallel$ $\overline{CB}$

Theorem 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



IF $\overline{DA} \parallel \overline{EB} \parallel \overline{FC}$

THEN $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$

or

$\frac{DE}{EF} = \frac{AB}{BC}$