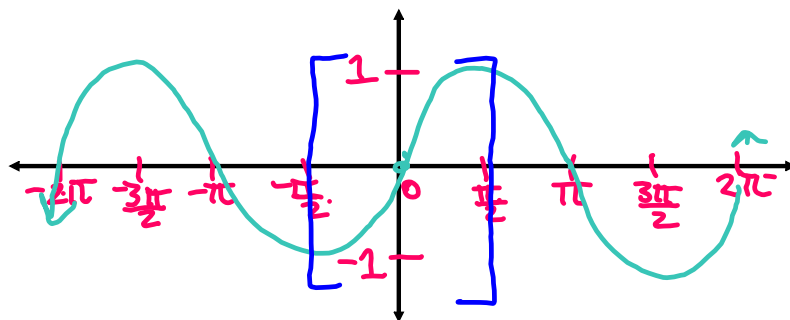


5.5 Inverse Trigonometric Functions

REMEMBER: An inverse **reverses** (undoes) a function.

Inverse Sine Function

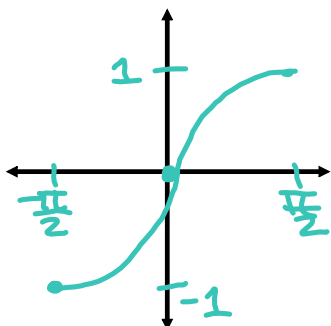
First, graph $y = \sin x$.



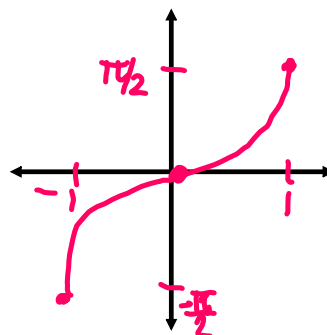
A function must pass the HLT in order for the inverse to be a function.

So we will restrict the **domain** of the sine graph to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
 QIV & QI

Now, graph $y = \sin x$ under these restrictions.



Then, graph $y = \sin^{-1}x$.



Graph $y = \sin^{-1}x$ by reflecting the graph of $y = \sin x$ in the line $y = x$.

Definition of the Inverse Sine Function

The **inverse sine function** is the function \sin^{-1} with domain $[-1, 1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

The inverse sine function is also called **arcsine** and is denoted **arcsin**.

$$\sin^{-1} x = \arcsin x$$

Evaluating the **Inverse Sine Function**
 $[-\frac{\pi}{2}, \frac{\pi}{2}]$
 QIV QI

Find $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

sin of ? equals $\frac{1}{2}$

Find $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$

sin of ? equals $-\frac{1}{2}$

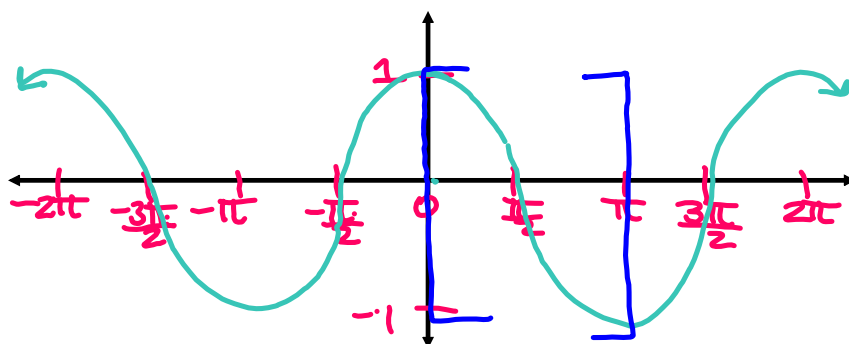
Use a calculator to find approximate values.

$$\sin^{-1}(.82) \approx .961$$

$$\sin^{-1}\left(\frac{1}{3}\right) \approx .340$$

Inverse Cosine Function

First, graph $y = \cos x$.

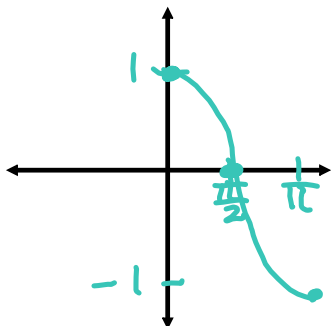


Just like $y = \sin x$, $y = \cos x$ is not one-to-one, so we must restrict the domain.

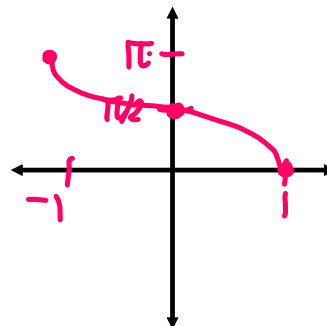
Where should we choose to restrict the domain?

So we will restrict the **domain** of the cosine graph to $[0, \pi]$.

Now, graph $y = \cos x$ under these restrictions.



Then, graph $y = \cos^{-1}x$.



Definition of the Inverse Cosine Function

The **inverse cosine function** is the function \cos^{-1} with domain $[-1, 1]$ and range $[0, \pi]$.

The inverse sine function is also called **arccosine** and is denoted **arccos**.

$$\cos^{-1} x = \arccos x$$

Evaluating the ^{QI QII}Inverse Cosine Function

Find $\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

cos of ? equals $\frac{\sqrt{3}}{2}$

Find $\cos^{-1}0 = \frac{\pi}{2}$

cos of ? equals 0

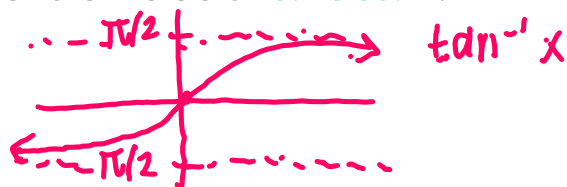
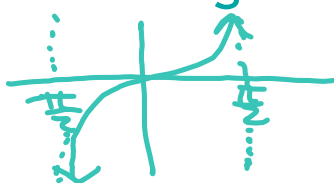
Find $\cos^{-1}(\frac{5}{7}) \approx .775$

Definition of the Inverse Tangent Function

We restrict the domain of the tangent function to the interval ^{QIV QI} $(-\frac{\pi}{2}, \frac{\pi}{2})$ in order to obtain a one-to-one function.

The **inverse tangent function** is the function \tan^{-1} with domain \mathbb{R} and range $(-\frac{\pi}{2}, \frac{\pi}{2})$.

The inverse tangent function is also called **arctangent** and is denoted **arctan**.



Evaluating the Inverse Tangent Function

Find $\tan^{-1}1. = \frac{\pi}{4}$
 tan of ? equals 1

Find $\tan^{-1}\sqrt{3}. = \frac{\pi}{3}$
 tan of ? equals $\sqrt{3}$


Find $\tan^{-1}(-20). \approx -1.521$
 tan of ? equals -20

Find the exact value of the expression,
 if it is defined.

$\cos(\sin^{-1}\frac{\sqrt{2}}{2}) = \cos(\frac{\pi}{4}) = \boxed{\frac{\sqrt{2}}{2}}$
 $\hookrightarrow \sin \text{ of } ? = \frac{\sqrt{2}}{2}$


~~$\cos(\cos^{-1}\frac{2}{3}) = \frac{2}{3}$~~
 $\hookrightarrow \cos \text{ of } ? = \frac{2}{3}$

Find the exact value of the expression,
if it is defined.

$$\sin(\sin^{-1} \frac{1}{4}) = \frac{1}{4}$$


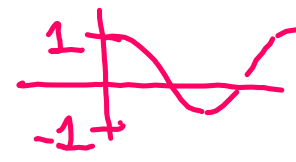
$$\tan(\tan^{-1} 5) = 5$$

tan of ? = 5



$$\cos(\cos^{-1} \frac{3}{2}) \rightarrow \text{undefined}$$

cos of ? = $\frac{3}{2}$



Find the exact value of the expression,
if it is defined.

$$\cos^{-1}(\cos \frac{5\pi}{4}) = \cos^{-1}(-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$$

QI QII
cos of ? = $-\frac{\sqrt{2}}{2}$

$$\sin^{-1}(\sin(-\frac{\pi}{6})) = \sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$$

QIV ~~QII~~

$$\tan^{-1}(\tan \frac{2\pi}{3}) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

~~QII~~ QIV

Find the exact value of the expression,
if it is defined.

$$\sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \frac{\pi}{6}$$

QI $\sin^{-1}\left(\frac{1}{2}\right)$
 QI \sin of ? = $\frac{1}{2}$

$$\cos\left(\sin^{-1} \frac{\sqrt{3}}{2}\right) = \frac{1}{2}$$

QII \sin of ? = $\frac{\sqrt{3}}{2}$
 $\cos \frac{\pi}{3}$

$$\tan\left(\sin^{-1} \frac{1}{2}\right) = \frac{\sqrt{3}}{3}$$

QI \sin of ? = $\frac{1}{2}$
 $\tan \frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$$\sin\left(\tan^{-1}(-1)\right) = -\frac{\sqrt{2}}{2}$$

QIV \tan ? = -1
 $\sin\left(-\frac{\pi}{4}\right)$