

Word Problems with Systems of Equations

Example 1

Lisa received a total score of 1340 on the SAT (verbal and math scores combined). Her math score was 400 points less than twice her verbal score. What was her math score and her verbal score?

$x = \text{math score}$ $y = \text{verbal score}$

$$\begin{array}{l}
 x + y = 1340 \longrightarrow 2y - 400 + y = 1340 \\
 x = 2y - 400 \\
 \begin{array}{r}
 x + 580 = 1340 \\
 -580 \quad -580 \\
 \hline
 x = 760
 \end{array} \\
 \begin{array}{r}
 3y - 400 = 1340 \\
 +400 \quad +400 \\
 \hline
 3y = 1740 \\
 \frac{3y}{3} = \frac{1740}{3} \\
 y = 580
 \end{array}
 \end{array}$$

760 math score
580 verbal score

Example 2

A Beach Resort is offering two weekend specials. One includes a 2-night stay with 4 meals and costs \$310. The other includes a 3-night stay with 7 meals and costs \$485. What is the cost of a 1-night stay? What is the cost per meal?

$x = \text{cost per 1-night stay}$ $y = \text{cost per meal}$

$$\begin{array}{l}
 -3 \cdot (2x + 4y) = (310) \cdot -3 \longrightarrow -6x - 12y = -930 \\
 2 \cdot (3x + 7y) = (485) \cdot 2 \longrightarrow 6x + 14y = 970
 \end{array}$$

$$\begin{array}{l}
 2x + 4(20) = 310 \\
 2x + 80 = 310 \\
 2x = 230 \\
 x = 115
 \end{array}$$

\$115/night
\$20/meal

$$\begin{array}{l}
 \frac{2y}{2} = \frac{40}{2} \\
 y = 20
 \end{array}$$

Example 3 $x = \begin{matrix} \# \text{ of students} \\ \text{in van} \end{matrix}$ $y = \begin{matrix} \# \text{ of students} \\ \text{in bus} \end{matrix}$

The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

$$\begin{aligned} 8x + 8y &= 240 \\ 4x + 1y &= 54 \end{aligned}$$

$$\begin{aligned} 8x + 8y &= 240 \\ -3x - 2y &= -108 \\ \hline 6y &= 132 \\ 6 & \quad 6 \\ \hline y &= 22 \end{aligned}$$

$$\begin{aligned} 4x + 22 &= 54 \\ 4x &= 32 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} 8x + 8y &= 240 \\ -32x - 8y &= -432 \\ \hline -24x &= -192 \\ -24 & \quad -24 \\ \hline x &= 8 \end{aligned}$$

$$\begin{aligned} x &= 8 \\ 32 + y &= 54 \\ y &= 22 \end{aligned}$$

8 students/
van
22 students/
bus

Example 4 $x = \begin{matrix} \text{price} \\ \# \text{ of senior tickets} \end{matrix}$ $y = \begin{matrix} \text{price} \\ \# \text{ of child tickets} \end{matrix}$

The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

$$\begin{aligned} 3x + y &= 38 \\ -3x + 2y &= -52 \\ \hline -y &= -14 \\ y &= 14 \end{aligned}$$

$$\begin{aligned} 3x + 14 &= 38 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

\$8/senior citizen
\$14/child