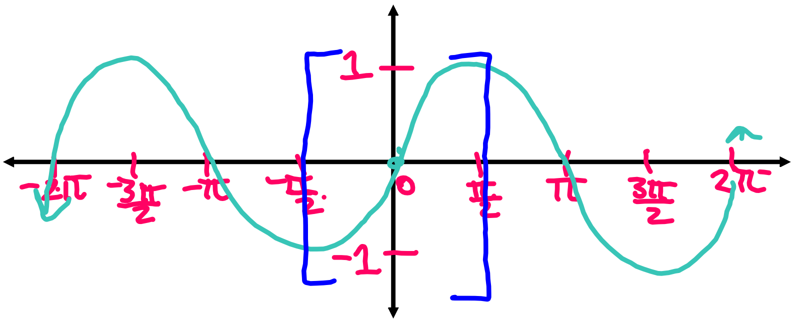
5.5 Inverse Trigonometric Functions

REMEMBER: An inverse reverses (undoes) a function.

Inverse Sine Function

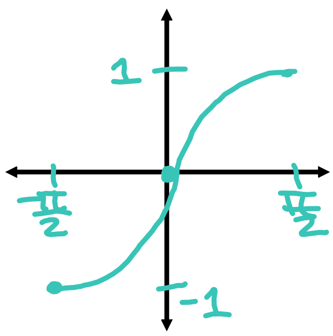
First, graph y = sin x.



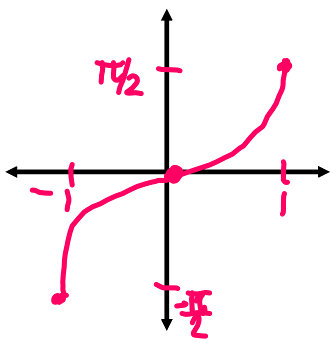
A function must pass the HLT in order for the inverse to be a function.

So we will restrict the domain of the sine graph to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
QIV & QI

Now, graph y = sin x under these restrictions.



Then, graph $y = \sin^{-1}x$.



Graph $y = \sin^{-1}x$ by reflecting the graph of y = sin x in the line y = x.

Definition of the Inverse Sine Function

The inverse sine function is the function $\sin^{-1}$ with domain [-1,1] and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

The inverse sine function is also called arcsine and is denoted arcsin.

$\sin^{-1} x = \arcsin x$

Evaluating the Inverse Sine Function

$[-\frac{\pi}{2}, \frac{\pi}{2}]$

QIV QI

Find $\sin^{-1}(\frac{1}{2})$. $= \frac{\pi}{6}$

sin of ? equals $\frac{1}{2}$

Find $\sin^{-1}(-\frac{1}{2})$. $= -\frac{\pi}{6}$

sin of ? equals $-\frac{1}{2}$

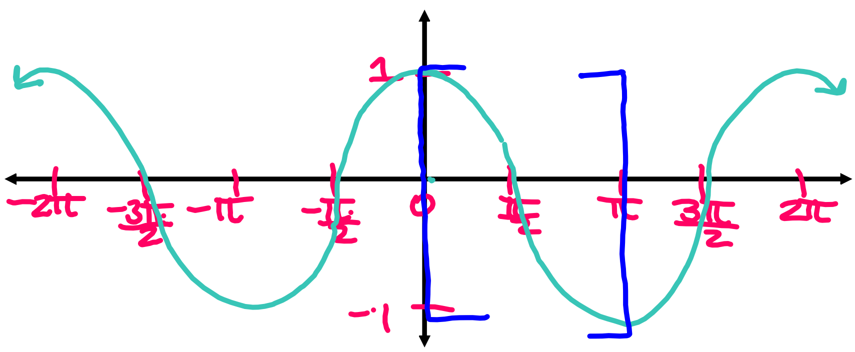
Use a calculator to find approximate values.

$\sin^{-1}(.82) \approx .961$

$\sin^{-1}(\frac{1}{3}) \approx .340$

Inverse Cosine Function

First, graph y = cos x.



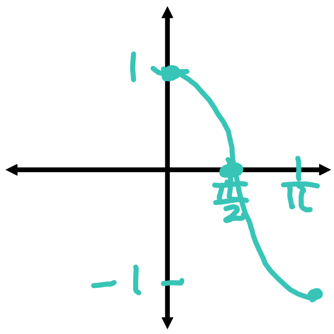
Just like y = sin x, y = cos x is not one-to-one, so we must restrict the domain.

Where should we choose to restrict the domain?

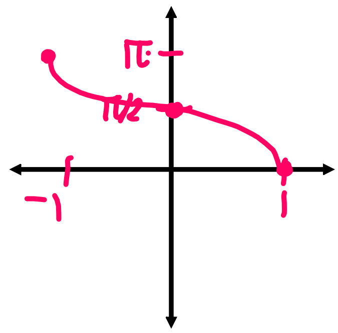
So we will restrict the domain of the cosine graph to $[0, \pi]$.

QI QII

Now, graph $y = \cos x$ under these restrictions.



Then, graph $y = \cos^{-1} x$.



Definition of the Inverse Cosine Function

The inverse cosine function is the function $\cos^{-1}$ with domain [-1,1] and range [0,π].

The inverse sine function is also called arccosine and is denoted arccos.

$\cos^{-1} x = \arccos x$

QI QII

Evaluating the Inverse Cosine Function

Find $\cos^{-1}(\frac{\sqrt{3}}{2})$. $= \frac{\pi}{6}$

cos of ? equals $\frac{\sqrt{3}}{2}$

Find $\cos^{-1}0$. $= \frac{\pi}{2}$

cos of ? equals 0

Find $\cos^{-1}(\frac{5}{7})$. $\approx .775$

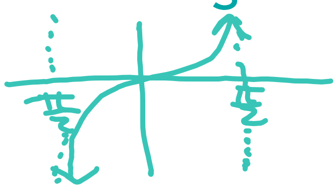
Definition of the Inverse Tangent Function

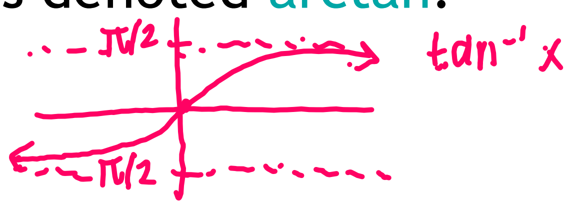
QIV QI

We restrict the domain of the tangent function to the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ in order to obtain a one-to-one function.

The inverse tangent function is the function $\tan^{-1}$ with domain $\mathbb{R}$ and range $(-\frac{\pi}{2}, \frac{\pi}{2})$.

The inverse tangent function is also called arctangent and is denoted arctan.





Evaluating the Inverse Tangent Function

Find $\tan^{-1} 1$. $= \frac{\pi}{4}$

tan of ? equals 1

Find $\tan^{-1}\sqrt{3}$. $= \frac{\pi}{3}$

tan of ? equals $\sqrt{3}$

Find $\tan^{-1}(-20)$. ≈ -1.521

tan of ? equals -20

Find the exact value of the expression, if it is defined.

$\cos\left(\underline{\sin^{-1}\frac{\sqrt{2}}{2}}\right) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

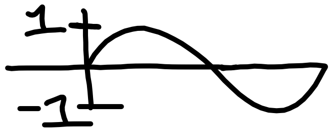
sin of ? $= \frac{\sqrt{2}}{2}$

$\cancel{\cos}\left(\cancel{\cos^{-1}}\frac{2}{3}\right) = \frac{2}{3}$

cos of ? $= \frac{2}{3}$

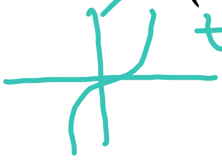
Find the exact value of the expression, if it is defined.

$\cancel{\sin(\sin^{-1}} \frac{1}{4}) = \frac{1}{4}$



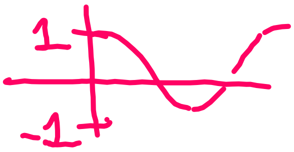
$\cancel{\tan(\tan^{-1}} 5) = 5$

tan of ? = 5



$\cos(\cos^{-1} \frac{3}{2}) \rightarrow$ undefined

cos of ? $= \frac{3}{2}$



Find the exact value of the expression, if it is defined.

QI, QII

$\cos^{-1}(\underline{\cos \frac{5\pi}{4}}) = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

cos of ? $= -\frac{\sqrt{2}}{2}$

QIV $\cancel{\text{QI}}$

$\sin^{-1}(\underline{\sin (-\frac{\pi}{6})}) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\cancel{\text{QI}}$ QIV

$\tan^{-1}(\tan \frac{2\pi}{3}) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

Find the exact value of the expression, if it is defined.

$\sin^{-1}(\sin \frac{5\pi}{6})$

$\cos(\sin^{-1} \frac{\sqrt{3}}{2})$

$\tan(\sin^{-1} \frac{1}{2})$

$\sin(\tan^{-1}(-1))$