8.2 Part 1 Rational Functions

Rational functions are used in science and engineering to model complex equations in areas such as

1) fields and forces in physics,
2) electronic circuitry,
3) aerodynamics,
4) medicine concentrations,
5) optics to improve image resolution, and
6) acoustics and sound.

Rational Functions and Their Graphs

- A rational function is the quotient of two polynomial functions.

- The equation of a rational function looks like

$$f(x) = \frac{p(x)}{q(x)}$$

where p(x) and q(x) are polynomials
AND q(x) is **NOT** zero.

EXAMPLE:

$$f(x) = \frac{2}{6+x}$$

p(x) is the numerator 2 AND
q(x) is the denominator (6 + x)

Which of the functions below are rational functions?

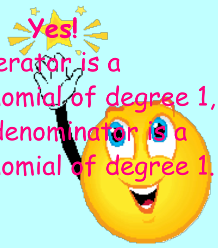
Drag each function to the shaded box below to check your answer!

$$y = \frac{x+2}{2x^2+3x-2} \qquad y = \frac{x^2+2}{|x|} \qquad y = \frac{3x}{x-4}$$

Yes!
Numerator is a polynomial of degree 1, and denominator is a polynomial of degree 2.

No!
Denominator is NOT a polynomial.

Yes!
Numerator is a polynomial of degree 1, and denominator is a polynomial of degree 1.



A. Domain of Rational Functions

D = all reals, EXCEPT..... any number that makes the denominator equal 0.

> To find the exceptions (also called excluded values) for the domain set the denominator equal to 0 and solve for the variable.

Examples: Find the domain of each function.

1. $f(x) = \dfrac{x}{2x-7}$

$$2x - 7 \neq 0$$
$$\quad +7 \quad +7$$
$$\frac{2x}{2} \neq \frac{7}{2}$$
$$x \neq \frac{7}{2}$$

Examples continued:
Find the domain of each rational function.

2. $f(x)=\dfrac{x}{(\underline{x-7})(\underline{x+3})}$

$x-7\neq 0$ $\quad$ $x+3\neq 0$

$x\neq 7$ $\quad$ $x\neq -3$

3. $y=\dfrac{x^2-2}{x^2-9x-36}$

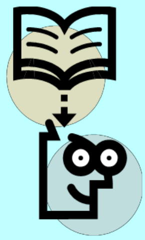
$x^2-9x-36\neq 0$

$(x-12)(x+3)\neq 0$

$x-12\neq 0$ $\quad$ $x+3\neq 0$

$x\neq 12$ $\quad$ $x\neq -3$

B. Asymptotes and Holes of Rational Functions



1. Vertical Asymptotes
   May occur at excluded values of the domain.

To find the vertical asymptotes (VA) of a rational function:
1. Factor numerator and denominator, if possible.
2. Identify factors of the denominator that are **NOT** factors of the numerator.
3. Set each identified factor equal to 0 and solve for the variable.

☆ These x-values are the vertical asymptotes!!!☆

Examples continued:
Find the domain of each rational function

4. $y = \dfrac{12-2x}{x^2-4}$

$x^2-4 \neq 0$

$(x-2)(x+2) \neq 0$

$x-2 \neq 0 \quad x+2 \neq 0$

$x \neq 2 \quad x \neq -2$

5. $f(x) = \dfrac{x^2-5x+4}{2x^2-7x-4}$

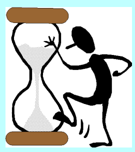
$2x^2-7x-4 \neq 0$

$(2x+1)(x-4) \neq 0$

$2x+1 \neq 0 \quad x-4 \neq 0$

$x \neq -\frac{1}{2} \quad x \neq 4$

Examples: Find the vertical asymptotes, if any.



1. $f(x) = \dfrac{2x}{x^2-1} = \dfrac{2x}{(x-1)(x+1)}$

$x-1=0$

$x+1=0$

VA: $x=1, \ x=-1$

2. $f(x) = \dfrac{3x}{x^2-3x+2} = \dfrac{3x}{(x-1)(x-2)}$

$x-1=0$

$x-2=0$

VA: $x=1, \ x=2$

3. $f(x) = \dfrac{2x+4}{x^2-3x-10} = \dfrac{2(x+2)}{(x-5)(x+2)}$ hole

$x-5=0$ VA: $x=5$

B. Asymptotes and Holes of Rational Functions (continued)

2. Holes

> If a factor of the denominator IS a factor of the numerator, then a hole in the graph occurs.

Examples: Find the values of x for any holes in the graph of each function.

1. $f(x) = \frac{2x}{x^2 - 4x} = \frac{2\cancel{x}}{\cancel{x}(x-4)} = \frac{2}{x-4}$

$x = 0$

$y = \frac{2}{0-4} = \frac{2}{-4}$

$-\frac{1}{2}$

Hole $(0, -\frac{1}{2})$

2. $f(x) = \frac{3x}{x^2 - 3x + 2} = \frac{3 \cdot x}{(x-1)(x-2)}$

no holes

$2x - 1 = 0 \quad x = \frac{1}{2}$

3. $f(x) = \frac{2x-1}{2x^2 + 5x - 3} = \frac{\cancel{2x-1}}{\cancel{(2x-1)}(x+3)} = \frac{1}{x+3}$ $\quad \frac{1}{\frac{1}{2}+3} = \frac{1}{\frac{7}{2}} = \frac{2}{7}$

Hole $(\frac{1}{2}, \frac{2}{7})$

Try these: Find the vertical asymptotes and holes, if any.

Drag each function to the shaded box below to check your answer!

$\frac{3x\cancel{(x+4)}}{(x+3)\cancel{(x+4)}} = \frac{3x}{x+3}$

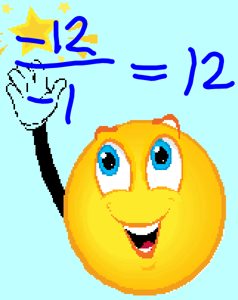
1. $f(x) = \frac{4x-3}{x^2 - 6x}$ 2. $f(x) = \frac{3x^2 + 12x}{x^2 + 7x + 12}$ 3. $f(x) = \frac{x + x^2}{x^3 - x}$

Hole: $x = -4$

$(-4, 12)$

$\frac{3 \cdot -4}{-4 + 3} = \frac{-12}{-1} = 12$

VA: $x = -3$



B. Asymptotes and Holes of Rational Functions (continued)

3. Horizontal Asymptotes (HA)

To find the horizontal asymptote (HA) of a rational function, you MUST compare the degree of the numerator to the degree of the denominator.

(Only look at the term with the largest exponent in both the numerator and denominator.)

3 Cases:

Let n = degree of numerator,
and let d = degree of denominator.

1. If $n < d$, then $y = 0$ is the HA.
2. If $n = d$, then $y = a/b$ is the HA ("a" and "b" are the coefficients of the leading terms in the numerator and denominator)
3. If $n > d$, then there is NO HA.

Examples: Find the horizontal asymptote, if any, of each function.

1. $y = \dfrac{x}{x^2 - 2x - 3}$ bottom heavy HA: $y=0$

2. $f(x) = \dfrac{2x+2}{3x-2}$ same HA: $y = \dfrac{2}{3}$

3. $f(x) = \dfrac{x^2-4}{x+5}$ top heavy no HA

Try these: Erase below problem to reveal answer !!!

Find the horizontal asymptote, if any, of each function.

1. $f(x)=\frac{-3}{x^2-3x}$ bottom heavy HA: $y=0$

2. $y=\frac{x+1}{1-x}$ same HA: $y=\frac{1}{-1} \rightarrow y=-1$

$\frac{x+1}{-x+1}$

3. $f(x)=\frac{x^2}{2x^2}$

Putting It ALL together!

Well.....almost all!

Find the domain, all asymptotes and holes.

1. $f(x)=\frac{3x-1}{x^2-9} = \frac{3x-1}{(x-3)(x+3)}$

D: $x-3\neq 0$ $x+3\neq 0$

$x\neq 3, -3$

VA: $x=3, -3$

Hole: none

Click to check your answers.

bottom heavy HA: $y=0$

Find the domain, all asymptotes and holes.

2. $f(x) = \frac{x^2 - 6x + 9}{x^2 - 5x + 6}$ $\boxed{\frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x-2)}}$

Click to check your answers.

D: $x \neq 3, 2$

VA: $x = 2$

Hole: $\frac{3-3}{3-2} = \frac{0}{1} = \overset{y}{0}$ $(3, 0)$

$x = 3$

Same HA: $y = 1$

C. Intercepts

Are we done yet??

1. Y-Intercept: substitute x = 0 into the equation.
   a. If "y" exists, this is the value of the y-intercept.
   b. If "y" is undefined, there is NO y-intercept.

2. X-Intercepts (if any): set the numerator equal to 0 and solve for the x values.

NOW!!!!

$y = x - 5$

Find the domain, all asymptotes, holes and intercepts.

3. $y = \frac{x^3 - 2x^2 - 15x}{x^2 + 3x}$

$\frac{\cancel{x}(x-5)\cancel{(x+3)}}{\cancel{x}\cancel{(x+3)}}$

Click to check your answers.

D: $x \neq -3, 0$

VA: none

Hole: $(0, -5)$ $(-3, -8)$

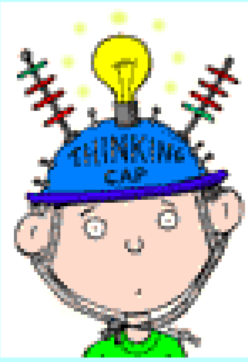
top HA: none

x-intercept: $(5, 0)$

$0 = x - 5$
$x = 5$

y-intercept: $(0, -5)$

$y = 0 - 5$
$y = -5$



Find the domain, all asymptotes, holes and intercepts.

4. $y = \frac{2x^2 - 1}{x^2 - 9}$

$y = \frac{2x^2 - 1}{(x+3)(x-3)}$

Click to check your answers.

D: $x \neq -3, 3$

VA: $x = -3, 3$

Hole: none

same HA: $y = 2$

x-intercept:

$x^2 - 9 \cdot 0 = \frac{2x^2 - 1}{x^2 - 9} \cdot x^2 - 9$

y-intercept:

Homework: Worksheet Practice 5-2 Rational Functions

(see attachments)

Attachments

Practice 8-2 Rational Functions.doc