6.4 Part 2 Solving Polynomial Equations

Factoring Refresher

- GCF
- Difference of Squares
- Difference of Cubes ~~binomials~~
- Sum of Cubes
- Sum/Product Chart ~~trinomials~~
- Grouping ~~4 terms~~

Solving Refresher

- Set each factor = 0
- Quadratics: x^2
  - Factor
  - > Complete the Square
  - > Quadratic Formula
  - > Take Square Roots

solutions = roots = x-intercepts = zeros

Example 1

Solve $2x^3 - 7x^2 + 3x = 0$.

$$x(2x^2 - 7x + 3) = 0$$

sum -7 product 6

$+6$ $(-1 \cdot -6)$

~~$2 \cdot 3$~~ ~~$2 \cdot 3$~~

$$\frac{-1}{2} \quad \frac{-6}{2} \quad \frac{-3}{1}$$

$$x(2x-1)(x-3) = 0$$

$\boxed{x=0}$

$2x - 1 = 0$
    $+1 \quad +1$
$\frac{2x}{2} = \frac{1}{2}$
$\boxed{x = \frac{1}{2}}$

$x - 3 = 0$
    $+3 \quad +3$
$\boxed{x = 3}$

Example 2

Solve $5x^3 + 4x = 12x^2$.

$$-12x^2 \qquad -12x^2$$

$$5x^3 - 12x^2 + 4x = 0$$

$$x(5x^2 - 12x + 4) = 0$$

sum -12 product 20

$$\frac{-2}{1} \quad \frac{-10}{5} \quad \frac{-2}{5}$$

$$\underline{x}(\underline{x-2})(\underline{5x-2}) = 0$$

$$\boxed{x=0} \qquad x-2=0 \qquad 5x-2=0$$
$$\qquad\qquad +2 \;\; +2 \qquad\quad +2 \;\; +2$$
$$\qquad\qquad \boxed{x=2} \qquad \frac{5x}{5} = \frac{2}{5}$$
$$\qquad\qquad\qquad\qquad\quad \boxed{x=\frac{2}{5}}$$

Example 3

Find all solutions of $3x^4 - 12x^2 + 9 = 0$.

$$3(x^4 - 4x^2 + 3) = 0$$

sum -4 product 3

$$\frac{-1}{1} \qquad \frac{-3}{1}$$

$$3(x^2 - 1)(x^2 - 3) = 0$$
$$\quad (x)^2 \;(1)^2$$

$$3(\underline{x-1})(\underline{x+1})(x^2-3) = 0$$

$$\cancel{3=0} \qquad x-1=0 \qquad x+1=0 \qquad x^2-3=0$$
$$\qquad\qquad\quad +1 \;\; +1 \qquad -1 \;\; -1 \qquad\quad +3 \;\; +3$$
$$\qquad\qquad\quad \boxed{x=1} \qquad \boxed{x=-1} \qquad \sqrt{x^2} = \sqrt{3}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \boxed{x = \pm\sqrt{3}}$$

Example 4

Find all roots of $x^4 + 7x^2 - 18 = 0$.

$$\underline{\text{sum } 7 \quad \text{product } -18}$$

$$\frac{9}{1} \qquad \frac{-2}{1}$$

$$(x^2 + 9)(x^2 - 2) = 0$$

$$x^2 + 9 = 0 \qquad\qquad x^2 - 2 = 0$$
$$\underline{\quad -9 \; -9\quad} \qquad\qquad \underline{\quad +2 \; +2\quad}$$
$$\sqrt{x^2} = \sqrt{-9} \qquad\qquad \sqrt{x^2} = \sqrt{2}$$

$$\boxed{x = \pm 3i} \qquad\qquad \boxed{x = \pm\sqrt{2}}$$

Example 5

Find all zeros of $x^3 - 3x^2 + 5x = 15$.

$$\underline{\quad\quad\quad -15 \quad -15\quad}$$

$$(x^3 - 3x^2) + 5x - 15) = 0$$

$$x^2(x - 3) + 5(x - 3) = 0$$

$$(x - 3)(x^2 + 5) = 0$$

$$x - 3 = 0 \qquad\qquad x^2 + 5 = 0$$
$$\underline{\quad +3 \; +3\quad} \qquad\qquad \underline{\quad -5 \quad -5\quad}$$

$$\boxed{x = 3} \qquad\qquad \sqrt{x^2} = \sqrt{-5}$$

$$\boxed{x = \pm i\sqrt{5}}$$

Example 6

Find all solutions of $2x^4 - 20x^2 = 0$.

$$2x^2(x^2 - 10) = 0$$

$$\frac{2 \cdot x^2}{2} = \frac{0}{2}$$

$$\sqrt{x^2} = \sqrt{0}$$

$$\boxed{x = 0}$$

$$x^2 - 10 = 0$$
$$ + 10 \quad\quad +10$$

$$\sqrt{x^2} = \sqrt{10}$$

$$\boxed{x = \pm\sqrt{10}}$$

Example 7

Find all roots of $x^3 - 8 = 0$.

$$(x)^3 \quad (2)^3$$

$$(a - b)(a^2 + ab + b^2)$$
$$(x - 2)(x^2 + x \cdot 2 + 2^2) = 0$$
$$(x - 2)(x^2 + 2x + 4) = 0$$

$$x - 2 = 0$$
$$+2 \quad +2$$

$$\boxed{x = 2}$$

$$x^2 + 2x + 4 = 0$$
$$a = 1 \quad b = 2 \quad c = 4$$
$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{-12}}{2}$$
$$x = \frac{-2 \pm 2i\sqrt{3}}{2}$$
$$x = \frac{-2}{2} \pm \frac{2i\sqrt{3}}{2}$$
$$\boxed{x = -1 \pm i\sqrt{3}}$$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$