

6.4 Part 2 Solving Polynomial Equations

Factoring Refresher

- GCF
 - Difference of Squares
 - Difference of Cubes
 - Sum of Cubes
 - Sum/Product Chart
 - Grouping
- binomials* (bracketed around Difference of Squares and Difference of Cubes)
- trinomials* (bracketed around Sum/Product Chart)
- 4 terms* (with arrow pointing to Grouping)

Solving Refresher

- Set each factor = 0
- Quadratics: x^2
- Factor
- > Complete the Square
- Quadratic Formula
- > Take Square Roots

solutions = roots = x-intercepts = zeros

Example 1

Solve $2x^3 - 7x^2 + 3x = 0$.

$$x(2x^2 - 7x + 3) = 0$$

sum -7 product 6

+	6	(-1, -6)
-	3	-2, -3

$\frac{-1}{2}$	$\frac{-6}{2}$	$\frac{-3}{1}$
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$$x(2x-1)(x-3) = 0$$

$$\boxed{x=0}$$

$$\begin{array}{r} 2x-1=0 \\ +1 \quad +1 \\ \hline 2x = 1 \\ \frac{2x}{2} = \frac{1}{2} \\ \hline \boxed{x = \frac{1}{2}} \end{array}$$

$$\begin{array}{r} x-3=0 \\ +3 \quad +3 \\ \hline \boxed{x=3} \end{array}$$

Example 2

Solve $5x^3 + 4x = 12x^2$.

$$\begin{array}{r} 5x^3 + 4x = 12x^2 \\ -12x^2 \quad -12x^2 \\ \hline 5x^3 - 12x^2 + 4x = 0 \\ x(5x^2 - 12x + 4) = 0 \end{array}$$

sum -12 product 20

$$\begin{array}{ccc} -2 & -10 & -2 \\ \hline 1 & 5 & 5 \end{array}$$

$$x(x-2)(5x-2) = 0$$

$$\boxed{x=0}$$

$$\begin{array}{r} x-2=0 \\ +2 \quad +2 \\ \hline \end{array}$$

$$\boxed{x=2}$$

$$\begin{array}{r} 5x-2=0 \\ +2 \quad +2 \\ \hline \end{array}$$

$$\frac{5x}{5} = \frac{2}{5}$$

$$\boxed{x = \frac{2}{5}}$$

Example 3Find all solutions of $3x^4 - 12x^2 + 9 = 0$.

$$3(x^4 - 4x^2 + 3) = 0$$

sum -4 product 3

$$\begin{array}{cc} -1 & -3 \\ \hline 1 & 1 \end{array}$$

$$3(x^2-1)(x^2-3) = 0$$

$$3(x-1)(x+1)(x^2-3) = 0$$

$$\cancel{3=0}$$

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$\boxed{x=1}$$

$$\begin{array}{r} x+1=0 \\ -1 \quad -1 \\ \hline \end{array}$$

$$\boxed{x=-1}$$

$$\begin{array}{r} x^2-3=0 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\sqrt{x^2} = \pm \sqrt{3}$$

$$\boxed{x = \pm \sqrt{3}}$$

Example 4Find all roots of $x^4 + 7x^2 - 18 = 0$.

Sum 7 product -18

$$\frac{9}{1} \quad \frac{-2}{1}$$

$$(x^2 + 9)(x^2 - 2) = 0$$

$$\begin{array}{r} x^2 + 9 = 0 \\ -9 \quad -9 \end{array}$$

$$\sqrt{x^2} = \sqrt{09}$$

$$x = \pm 3i$$

$$\begin{array}{r} x^2 - 2 = 0 \\ +2 \quad +2 \end{array}$$

$$\sqrt{x^2} = \sqrt{2}$$

$$x = \pm \sqrt{2}$$

Example 5Find all zeros of $x^3 - 3x^2 + 5x - 15 = 0$.

$$\begin{array}{r} x^3 - 3x^2 + 5x - 15 = 0 \\ -15 \quad -15 \end{array}$$

$$(x^3 - 3x^2) + (5x - 15) = 0$$

$$x^2(x - 3) + 5(x - 3) = 0$$

$$(x - 3)(x^2 + 5) = 0$$

$$\begin{array}{r} x - 3 = 0 \\ +3 \quad +3 \end{array}$$

$$x = 3$$

$$\begin{array}{r} x^2 + 5 = 0 \\ -5 \quad -5 \end{array}$$

$$\sqrt{x^2} = \sqrt{05}$$

$$x = \pm i\sqrt{5}$$

Example 6Find all solutions of $2x^4 - 20x^2 = 0$.

$$\underline{2x^2}(\underline{x^2 - 10}) = 0$$

$$\frac{2 \cdot x^2}{2} = \frac{0}{2}$$

$$\sqrt{x^2} = \sqrt{0}$$

$$x = 0$$

$$\begin{array}{r} x^2 - 10 = 0 \\ +10 \quad +10 \\ \hline \end{array}$$

$$\sqrt{x^2} = \sqrt{10}$$

$$x = \pm\sqrt{10}$$

Example 7Find all roots of $x^3 - 8 = 0$.

$$(x)^3 (2)^3$$

$$(a - b)(a^2 + ab + b^2)$$

$$(x - 2)(x^2 + x \cdot 2 + 2^2) = 0$$

$$\underline{(x - 2)}(\underline{x^2 + 2x + 4}) = 0$$

$$\begin{array}{r} x - 2 = 0 \\ +2 \quad +2 \\ \hline \end{array}$$

$$x = 2$$

$$x^2 + 2x + 4 = 0$$

$$a = 1 \quad b = 2 \quad c = 4$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{0 - 12}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$x = \frac{-2}{2} \pm \frac{2i\sqrt{3}}{2}$$

$$x = -1 \pm i\sqrt{3}$$

$$\begin{array}{r} 2 \overline{) 12} \\ \underline{2} \quad 6 \\ \underline{6} \quad 0 \\ 0 \end{array}$$