8.1 Direct, Inverse, Joint, and Combined Variation

RECALL from Algebra 1: Direct Variation

If y varies directly as x, then y = kx.

constant of variation

Example 1

The variable y varies directly as x, $y = kx$

and $y = 6$ when $x = 2.5$.

a) Find the constant of variation. $\frac{6}{2.5} = \frac{k \cdot 2.5}{2.5}$ $k = 2.4$

b) Write the appropriate direct variation equation. $y = 2.4x$

c) Find y when x is 0.5, 1, and 1.5.

$y = 2.4(0.5)$ $\boxed{y = 1.2}$

$y = 2.4(1)$ $\boxed{y = 2.4}$

$y = 2.4(1.5)$ $\boxed{y = 3.6}$

Inverse Variation

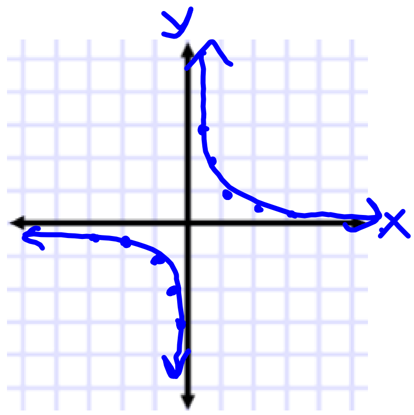
If y varies inversely as x, then $y = \frac{k}{x}$.

Example 1

Graph $y = \frac{1}{x}$.

x	y
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$\cancel{0}$	
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3

x	y
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
$\frac{1}{2}$	2
$\frac{1}{3}$	3



Example 2

The variable y varies inversely as x, and y = 6 when x = 2.5.

a) Find the constant of variation.

b) Write the appropriate inverse variation equation.

c) Find y when x is 0.5, 1, and 1.5.

$y = \frac{k}{x}$

$2.5 \cdot 6 = \frac{k}{\cancel{2.5}} \cdot \cancel{2.5}$

$k = 15$

$y = \frac{15}{x}$

$y = \frac{15}{0.5}$ $\qquad$ $y = \frac{15}{1}$ $\qquad$ $y = \frac{15}{1.5}$

$\boxed{y = 30}$ $\qquad$ $\boxed{y = 15}$ $\qquad$ $\boxed{y = 10}$

Joint Variation

If y varies jointly as x and z, then y = kxz.

Example 3

The variable y varies jointly as x and z, and y = 16 when x = 4 and z = $\frac{1}{2}$.

a) Find the constant of variation.

b) Write the appropriate joint variation equation.

c) Find y when x = 2 and z = $\frac{1}{4}$.

$y = kxz$

$16 = k \cdot 4 \cdot \frac{1}{2}$

$\frac{16}{2} = \frac{k \cdot 2}{2}$

$8 = k$

$y = 8xz$

$y = 8 \cdot 2 \cdot \frac{1}{4}$

$\boxed{y = 4}$

Combined Variation

Combined variation is any combination of direct, inverse, and/or joint variation.

Ex: *b varies jointly as c and e and inversely as d*

mult. div.

$$b = \frac{kce}{d}$$

Example 4

Write a general equation for each.

a) h varies jointly as m and n and inversely as p

mult. div.

$h = \frac{kmn}{p}$

b) j varies directly as the square of f and inversely as t

mult k^2 div

$j = \frac{kf^2}{t}$

c) q varies directly as w and inversely as the cube root of b

$q = \frac{kw}{\sqrt[3]{b}}$

Example 5

The variable y varies jointly as x and z, and inversely as w. If y = 72 when x = 6, z = 3, and w = $\frac{1}{2}$:

a) Find the constant of variation. $k = 2$

b) Write the appropriate combined variation equation. $\boxed{y = \frac{2xz}{w}}$

c) Find y when w = 18, x = $\frac{1}{4}$, and z = 12.

$y = \frac{kxz}{w}$

$72 = \frac{k \cdot 6 \cdot 3}{\frac{1}{2}}$

$72 = \frac{k \cdot 18}{\frac{1}{2}}$

$72 = k \cdot 18 \cdot 2$

$\frac{72}{36} = \frac{k \cdot 36}{36}$

$\boxed{2 = k}$

$y = \frac{2 \cdot \frac{1}{4} \cdot 12}{18}$

$y = \frac{6}{18}$

c) $\boxed{y = \frac{1}{3}}$

Example 6

In a local school, the number of girls (g) varies directly as the number of boys (b) and inversely as the number of teachers (t). When there were 50 girls, there were 20 teachers and 10 boys. How many boys were there when there were 10 girls and 100 teachers?

$\boxed{10 \text{ boys}}$

$$g = \frac{kb}{t} \longrightarrow g = \frac{100b}{t}$$

$$20 \cdot 50 = \frac{k \cdot 10}{\cancel{20}} \cdot \cancel{20}$$

$$\frac{1000}{10} = \frac{k \cdot 10}{10}$$

$$100 = k$$

$$100 \cdot 10 = \frac{100b}{100} \cdot 100$$

$$\frac{1000}{100} = \frac{100b}{100}$$

$$10 = b$$

Example 7

Cheers (c) at a sporting event varied jointly as the number of fans (f) and the square of the jubilation factor (j). When there were 100 fans and the jubilation factor was 4, there were 1000 cheers. How many cheers were there when there were only 10 fans whose jubilation factor was 20?

$$c = kfj^2 \longrightarrow c = \frac{5}{8} f j^2$$

$$1000 = k \cdot 100 \cdot \cancel{4^2}^{16}$$

$$\frac{1000}{1600} = \frac{k \cdot 1600}{1600}$$

$$\frac{5}{8} = k$$

$$c = \frac{5}{8} \cdot 10 \cdot \cancel{20^2}^{400}$$

$$c = \frac{5}{\cancel{8}} \cdot \cancel{4000}^{500}$$

$$c = 2500$$

$\boxed{2500 \text{ cheers}}$