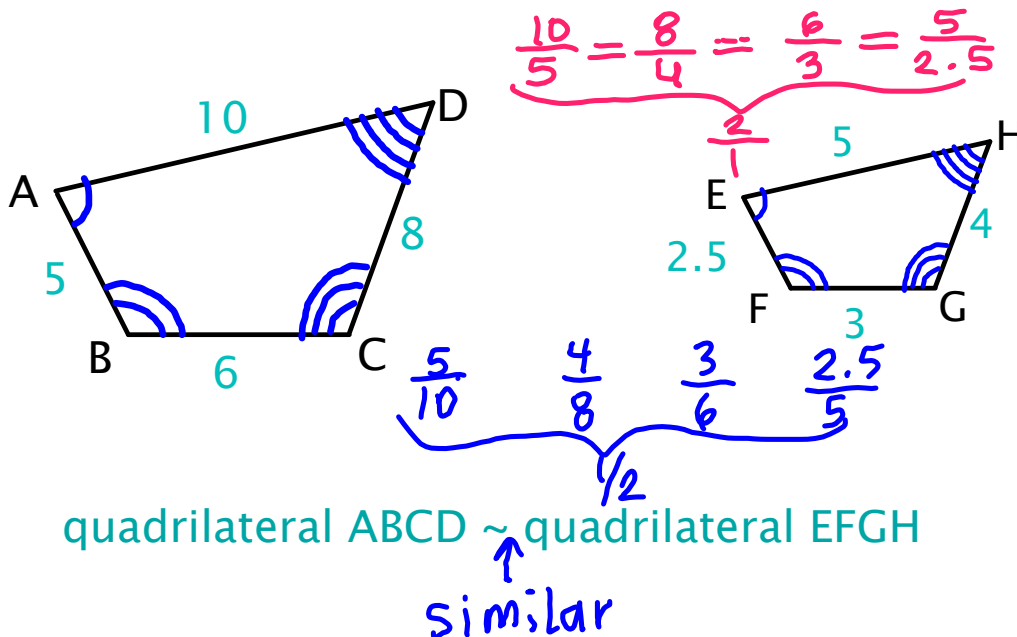Charles Demuth's painting   The Figure 5 in Gold (1928)
inspired by William Carlos Williams' poem: The Great Figure
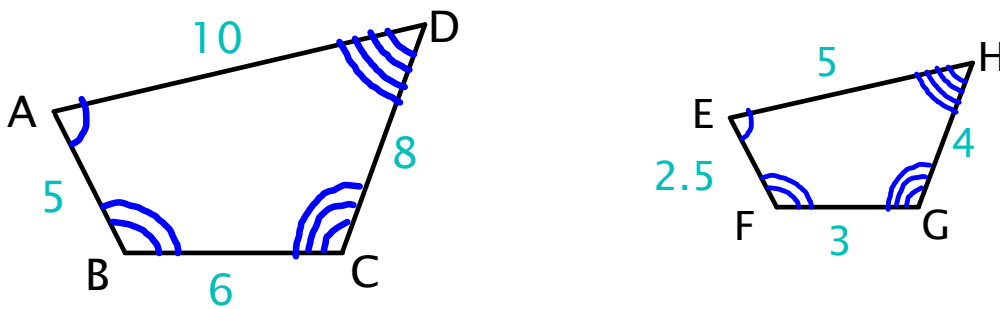

The Great Figure
Among the rain
and lights
I saw the figure 5
in gold
on a red
fire truck
moving
tense
unheeded
to gong clangs
siren howls
and wheels rumbling
through the dark city

6.1 PART 3 DEFINITION OF SIMILAR POLYGONS

Two polygons are similar if and only if their
corresponding angles are <u>congruent</u> and the
corresponding sides lengths are <u>proportional</u>.

$$\frac{10}{5} = \frac{8}{4} = \frac{6}{3} = \frac{5}{2.5}$$

$$2$$

$$\frac{5}{10} \quad \frac{4}{8} \quad \frac{3}{6} \quad \frac{2.5}{5}$$

$$1/2$$

quadrilateral ABCD ~ quadrilateral EFGH

↑
similar

Quadrilateral ABCD with: AD = 10, DC = 8, BC = 6, AB = 5

Quadrilateral EFGH with: EH = 5, HG = 4, FG = 3, EF = 2.5

We can make the following statements about these quadrilaterals...

$$\frac{10}{5} = \frac{8}{4} = \frac{6}{3} = \frac{5}{2.5} \longrightarrow \frac{2}{1}$$

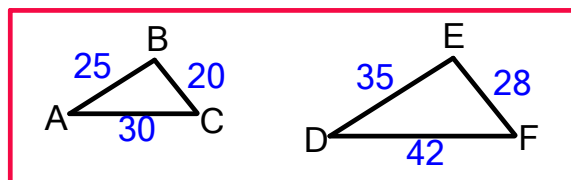$$\frac{5}{10} \quad \frac{4}{8} \quad \frac{3}{6} \quad \frac{2.5}{5} \longrightarrow \frac{1}{2}$$

<u>scale factor</u>: the ratio of the lengths of two corresponding sides to two similar polygons

EXAMPLE 1

$\triangle ABC \sim \triangle DEF.$

Triangle with B top, A bottom-left, C bottom-right: AB = 25, BC = 20, AC = 30

Triangle with E top, D bottom-left, F bottom-right: DE = 35, EF = 28, DF = 42

a) List all pairs of congruent angles.

$$\angle A \cong \angle D, \quad \angle B \cong \angle E, \quad \angle C \cong \angle F$$

b) Check that the ratios of corresponding side lengths are equal.

$$\frac{25}{35} = \frac{20}{28} = \frac{30}{42} \checkmark$$

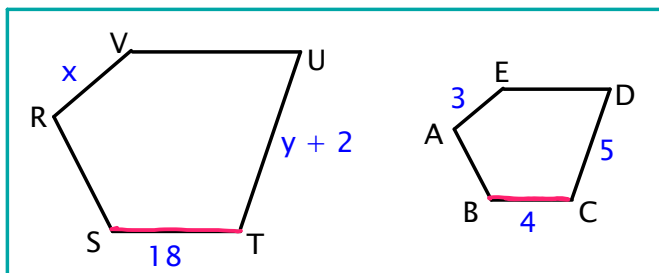$$\frac{5}{7} = \frac{5}{7} = \frac{5}{7}$$

c) Write the ratios of the corresponding side lengths in a statement of proportionality.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

EXAMPLE 2

Polygon RSTUV is similar to polygon ABCDE.

a) Find the scale factor of polygon RSTUV to polygon ABCDE.   big : small

$$\frac{18}{4} \longrightarrow \boxed{\frac{9}{2}}$$

b) Find the values of x and y.

$$\frac{9}{2} = \frac{x}{3}$$

$$\frac{2x}{2} = \frac{27}{2}$$

$$\boxed{x = \frac{27}{2}}$$

$$\frac{9}{2} = \frac{y+2}{5}$$

$$2(y+2) = 45$$

$$2y + 4 = 45$$

$$\underline{-4 \qquad -4}$$

$$\frac{2y}{2} = \frac{41}{2}$$

$$\boxed{y = \frac{41}{2}}$$

EXAMPLE 3

PQRS ~ UTWV.
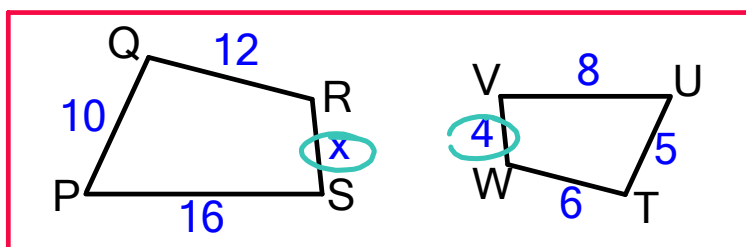
Small to big

a) What is the scale factor of UTWV to PQRS?

$$\frac{6}{12} \longrightarrow \boxed{\frac{1}{2}}$$

b) What is the scale factor of PQRS to UTWV?   big to small
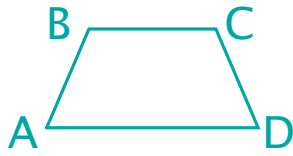
$$\boxed{\frac{2}{1}}$$

c) Find the value of x.

$$\frac{2}{1} = \frac{x}{4} \qquad \text{or} \qquad \frac{1}{2} = \frac{4}{x}$$

$$\boxed{x = 8}$$

Theorem 6.1: Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters
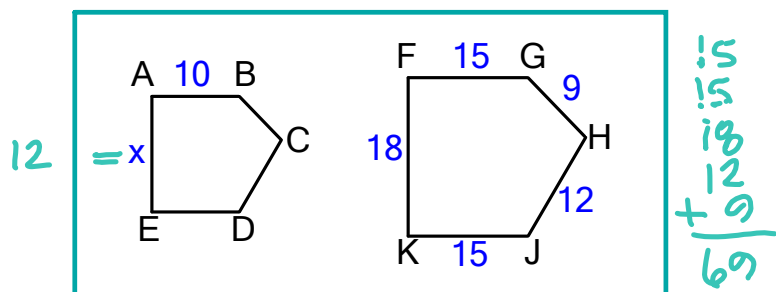is equal to the ratios of their corresponding sides lengths.

B — C   X — Y

A — D   W — Z

If ABCD ~ WXYZ, then...

$$\frac{AB+BC+CD+DA}{WX+XY+YZ+ZW} = \frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW}$$

EXAMPLE 4

ABCDE ~ FGHJK.

A 10 B   F   15   G
12 = x   C   9
E   D   18   H
              K   15   J   12

$$\begin{array}{r} 15 \\ 15 \\ 18 \\ 12 \\ + 9 \\ \hline 69 \end{array}$$

a) Find the scale factor of FGHJK to ABCDE.

big   small

$$\frac{15}{10} \longrightarrow \boxed{\frac{3}{2}}$$
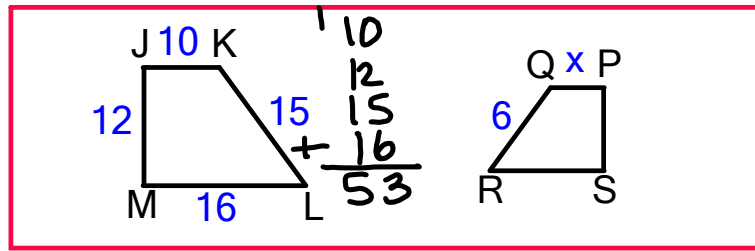
b) Find the value of x.

$$\frac{3}{2} = \frac{18}{x} \qquad \frac{3x}{3} = \frac{36}{3} \longrightarrow \boxed{x = 12}$$

c) Find the perimeter of ABCDE.

$$\frac{3}{2} = \frac{69}{p} \qquad \frac{3p}{3} = \frac{138}{3} \longrightarrow \boxed{p = 46}$$

<u>EXAMPLE 5</u>

JKLM ~ PQRS.

Inside box (handwritten dimensions):

$$10$$
$$12$$
$$15$$
$$+\ 16$$
$$\overline{53}$$

a) What is the scale factor of JKLM to PQRS?

big   small

$$\frac{15}{6} \longrightarrow \boxed{\frac{5}{2}}$$

b) What is the scale factor of PQRS to JKLM?

small   big

$$\boxed{\frac{2}{5}}$$

c) Find the value of x.

$$\frac{5}{2} = \frac{10}{x} \qquad 5x = 20 \longrightarrow \boxed{x = 4}$$

d) Find the perimeter of PQRS. $\frac{5}{2} = \frac{53}{p} \qquad 5p = 106 \longrightarrow \boxed{p = \frac{106}{5}}$