# 6.7 SOLVING EQUATIONS AND MODELING

## RICHTER SCALE RATINGS

**The Richter magnitude scale was developed in 1935 by Charles F. Richter of the California Institute of Technology as a mathematical device to compare the size of earthquakes. The magnitude of an earthquake is determined from the logarithm of the amplitude of waves recorded by seismographs. Because of the logarithmic basis of the scale, each whole number increase in magnitude represents a tenfold increase in measured amplitude. As an estimate of energy, each whole number step in the magnitude scale corresponds to the release of about 31 times more energy than the amount associated with the preceding whole number value.**

## RICHTER SCALE RATINGS

| Richter Magnitudes | Earthquake Effects | Frequency of Occurrence |
|---|---|---|
| Less than 2.0 | Microearthquakes, not felt. | About 8,000 per day |
| 2.0-2.9 | Generally not felt, but recorded. | About 1,000 per day |
| 3.0-3.9 | Often felt, but rarely causes damage. | 49,000 per year (est.) |
| 4.0-4.9 | Noticeable shaking of indoor items, rattling noises. Significant damage unlikely. | 6,200 per year (est.) |
| 5.0-5.9 | Can cause major damage to poorly constructed buildings over small regions. At most slight damage to well-designed buildings. | 800 per year |
| 6.0-6.9 | Can be destructive in areas up to about 100 miles across in populated areas. | 120 per year |
| 7.0-7.9 | Can cause serious damage over larger areas. | 18 per year |
| 8.0-8.9 | Can cause serious damage in areas several hundred miles across. | 1 per year |
| 9.0-9.9 | Devastating in areas several thousand miles across. | 1 per 20 years |
| 10.0+ | Never recorded; see below for equivalent seismic energy yield. | Extremely rare (Unknown) |

$$M = \frac{2}{3}\log\frac{E}{10^{11.8}}$$

M is the magnitude

E is amount of energy (measured by ergs)
released by the earthquake

## EXAMPLES

1. One of the strongest earthquakes in recent history occurred in Mexico City in 1985 and measured 8.1 on the Richter scale. Find the amount of energy, E, released by this earthquake.

M ?

$$M = \frac{2}{3}\log\frac{E}{10^{11.8}}$$

$$\frac{3}{2}\cdot 8.1 = \cancel{\frac{2}{3}}\log\frac{E}{10^{11.8}}\cdot\cancel{\frac{3}{2}}$$

$$12.15 = \log\frac{E}{10^{11.8}}$$

$$10^{12.15} = \cancel{10^{\log}}\frac{E}{10^{11.8}}$$

$$10^{11.8}\cdot 10^{12.15} = \frac{E}{\cancel{10^{11.8}}}\cdot\cancel{10^{11.8}}$$

$$\boxed{8.913\times 10^{23}\text{ ergs}\approx E}$$

## EXAMPLES

2. An earthquake occurred in the Philippines on February 10, 2017. It measured 6.5 on the Richter scale. Find the amount of energy released by the earthquake.

$$\frac{3}{2}\cdot 6.5 = \cancel{\frac{2}{3}}\log\frac{E}{10^{11.8}}\cdot\cancel{\frac{3}{2}}$$

$$9.75 = \log\frac{E}{10^{11.8}}$$

$$10^{9.75} = \cancel{10^{\log}}\frac{E}{10^{11.8}}$$

$$10^{11.8}\cdot 10^{9.75} = \frac{E}{\cancel{10^{11.8}}}\cdot\cancel{10^{11.8}}$$

$$\boxed{3.548\times 10^{21}\text{ ergs}\approx E}$$

## EXAMPLES

3. On January 17, 1995, an earthquake struck Osaka, Kyoto, and Kobe, Japan, injuring more than 36,000 people and causing an estimated $100 billion of damage. The quake released about $3.98 \times 10^{22}$ ergs of energy. E

Find the earthquake's magnitude on the Richter scale. Round to the nearest tenth.

$$M = \frac{2}{3}\log\frac{3.98 \times 10^{22}}{10^{11.8}}$$

$$\boxed{M \approx 7.2}$$

## EXAMPLES

4. A population of bacteria grows exponentially. A population that initially consists of 10,000 bacteria grows to 25,000 after 2 hours.

   a) Use the exponential growth function, $P(t) = P_0e^{kt}$, to find the value of k. Then write a function for this population of bacteria in terms of time, t. Round the value of k to the nearest hundredth.

$$\frac{25{,}000}{10{,}000} = \frac{10{,}000e^{2k}}{10{,}000}$$

$$\ln 2.5 = \cancel{\ln e}^{2k}$$

$$\frac{\ln 2.5}{2} = \frac{\cancel{2}k}{\cancel{2}}$$

$$\boxed{k \approx .46}$$

$$\boxed{P(t) = 10{,}000e^{.46t}}$$

## EXAMPLES

### 4. CONTINUED

b) How many bacteria will the population consist of after 12 hours, rounded to the nearest hundred thousand?

$P(12) = 10{,}000e^{.96(12)}$

$P(12) = 2496350.372$ bacteria

c) How many bacteria will the population consist of after 24 hours, rounded to the nearest hundred thousand?

$P(24) = 10{,}000e^{.46(24)}$

$P(24) = 623{,}176{,}517.9$ bacteria

## EXAMPLES

### 5. Solve for E.

$$\frac{3}{2} \cdot M = \frac{2}{3}\log\frac{E}{10^{11.8}} \cdot \frac{3}{2}$$

$$\frac{3}{2}M = \log\frac{E}{10^{11.8}}$$

$$10^{\frac{3}{2}M} = 10^{\cancel{\log}\frac{E}{10^{11.8}}}$$

$$10^{11.8} \cdot 10^{\frac{3}{2}M} = \frac{E}{\cancel{10^{11.8}}} \cdot \cancel{10^{11.8}}$$

$$\boxed{10^{\frac{3}{2}M+11.8} = E}$$

## EXAMPLES

6. Solve $\underbrace{\log x + \log (x - 3)}$ = 1 for x.

$$\log_{10} x(x-3) = 1$$

$$10^1 = x(x-3)$$

$$10 = x^2 - 3x$$

$$-10 \qquad -10$$

$$0 = x^2 - 3x - 10$$

$$0 = (x-5)(x+2)$$

$$\boxed{x=5} \qquad x \neq -2$$

## EXAMPLES

7. Solve $\underbrace{\log (x + 48) + \log x}$ = 2 for x.

$$\log_{10} x(x+48) = 2$$

$$10^2 = x(x+48)$$

$$100 = x^2 + 48x$$

$$0 = x^2 + 48x - 100$$

$$0 = (x+50)(x-2)$$

$$x \neq -50 \qquad \boxed{x=2}$$

## EXAMPLES

8. Solve $4e^{3x-5} = 72.$

$$\frac{\cancel{4}e^{3x-5}}{\cancel{4}} = \frac{72}{4}$$

$$\cancel{\ln e}^{3x-5} = \ln 18$$

$$3x - \cancel{5} = \ln 18$$
$$\underline{\ \ +\cancel{5} \qquad +5}$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{\ln 18 + 5}{3}$$

$$\boxed{x \approx 2.630}$$

## EXAMPLES

9. Solve $8e^{2x+5} = 56.$

$$\frac{\cancel{8}e^{2x+5}}{\cancel{8}} = \frac{56}{8}$$

$$\cancel{\ln e}^{2x+5} = \ln 7$$

$$2x + \cancel{5} = \ln 7$$
$$\underline{\ \ -\cancel{5} \qquad -5}$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{\ln 7 - 5}{2}$$

$$\boxed{x \approx -1.527}$$