8.1 Ratio and Proportion

**ratio**– a comparison of two quantities

\[
\begin{align*}
\frac{2}{3} & \quad 2:3 & \quad 2 \text{ to } 3
\end{align*}
\]

**Example 1**

Simplify each ratio.

a) \(\frac{6 \text{ in}}{2 \text{ ft}}\)

\[
\frac{6 \text{ in}}{24 \text{ in}} = \frac{1}{4}
\]

b) \(\frac{50 \text{ cm}}{2 \text{ m}}\)

\[
\frac{50 \text{ cm}}{200 \text{ cm}} = \frac{1}{4}
\]

**Example 2**

Simplify each ratio.

a) \(\frac{4 \text{ ft}}{3 \text{ yd}}\)

\[
\frac{4 \text{ ft}}{9 \text{ ft}} = \frac{4}{9}
\]

b) \(\frac{2 \text{ km}}{800 \text{ m}}\)

\[
\frac{2000 \text{ m}}{800 \text{ m}} = \frac{5}{2}
\]
Example 3

The perimeter of rectangle ABCD is 60 cm. The ratio of AB:BC is 3:2. Find the length and width of the rectangle.

\[ P = 2l + 2w \]

\[ 60 = 2(3x) + 2(2x) \]
\[ 60 = 6x + 4x \]
\[ 60 = 10x \]
\[ \frac{60}{10} = \frac{10x}{10} \]
\[ 6 = x \]

Example 4

The perimeter of the isosceles triangle shown is 56 in. The ratio of LM:MN is 5:4. Find the lengths of all sides of the triangle.

\[ P = LN + NM + LM \]
\[ 56 = 5x + 4x + 5x \]
\[ 56 = 14x \]
\[ \frac{56}{14} = \frac{14x}{14} \]
\[ 4 = x \]

LN = 5(4) = 20 in
NM = 4(4) = 16 in
LM = 5(4) = 20 in
Example 5
The ratio of the measures of the angles of a triangle are 1:2:3. Find the measures of the angles.

\[1x : 2x : 3x\]

\[180 = 1x + 2x + 3x\]
\[180 = 6x\]
\[30 = x\]

\[30^\circ, 60^\circ, 90^\circ\]

Example 6
The ratio of the measures of the angles of a triangle are 3:4:8. Find the measures of the angles.

\[180 = 3x + 4x + 8x\]
\[180 = \frac{15x}{15}\]
\[12 = x\]

\[36^\circ, 48^\circ, 96^\circ\]
An equation that equates two ratios is a proportion. To solve proportions, you will cross multiply.

**Example 7**

Solve each proportion.

a) $\frac{9}{14} = \frac{6}{x}$

$9x = 84$

$x = \frac{84}{9}$

$x = \frac{28}{3}$

b) $\frac{3}{y+2} = \frac{2}{y}$

$3y = 2(y+2)$

$3y = 2y + 4$

$-2y = -2$

$y = 4$

**Example 8**

Solve each proportion.

a) $\frac{3-x}{6} = \frac{x}{2}$

$6x = 6 - 2x + 2x$

$8x = 6$

$x = \frac{3}{4}$

b) $\frac{4w - 1}{2w - 3} = \frac{2}{3}$

$12w - 3 = 4w - 6$

$-4w = -3$

$8w = -3$

$W = -\frac{3}{8}$
Example 9
A diagram measuring 20 cm long is reduced on a copy machine to 15 cm long. If the width of the original copy is 16 cm, what is the width of the reduced copy?

\[
\frac{20}{16} = \frac{15}{w} \\
240 = 20w \\
\frac{240}{20} = \frac{20w}{20} \\
12 \text{ cm} = w
\]

Example 10
In a photograph taken from an airplane, a section of a city street is 3.5 inches long and \(\frac{1}{8}\) of an inch wide. If the actual street is 30 feet wide, how long is it?

\[
\frac{3.5}{\frac{1}{8}} = \frac{x}{30} \\
8 \cdot \frac{1}{8}x = 105 \cdot \frac{1}{8} \\
x = 840 \text{ ft}
\]
Example 11
Lee is reading a 374-page novel. It takes her 6 days to read the first 132 pages. At this rate, how many more days will it take her to finish the novel?

\[
\frac{6 \text{ days}}{132 \text{ pages}} = \frac{x}{242 \text{ pages}}
\]

\[
132x = 1452
\]

\[
x = \frac{1452}{132} = 11 \text{ days}
\]

Example 12
The ratio of an object's weight on Earth to its weight on the moon is 6:1. The first person to walk on the moon was Neil Armstrong. He weighted 165 pounds on Earth. What was his weight on the moon?

\[
\frac{\text{Earth}}{\text{Moon}} = \frac{165}{x}
\]

\[
6x = 165
\]

\[
x = \frac{165}{6} = 27.5 \text{ lb on moon}
\]