# 6.6 Part 2 The Natural Logarithm

*Recall that we learned about $e^x$, the natural exponential function.*

The natural logarithm is a logarithm with a base of e.

$$\log_e x$$

$$\Big\downarrow \text{ written without a base}$$

$$\ln x$$

## Example 1

Evaluate each using a calculator.

a) ln 8 b) ln 0.04 c) ln (-5) d) ln 100

2.079 -3.219 undefined 4.605

*Recall that logarithms are inverses of exponential functions.*

So the inverse of $e^x$ is ln x.

This means $\cancel{\ln e}^x$ = ___x___ and $\cancel{e}^{\ln x}$ = ___x___.

## Example 2

Simplify each expression.

a) $\cancel{e}^{\ln 9}$ b) $\cancel{\ln e}^{x/2}$ c) $\cancel{e}^{\ln 20}$ d) $3\cancel{\ln e}^4$

$9$ $\frac{x}{2}$ $20$ $3 \cdot 4$

$12$

*The properties of logarithms apply to the natural logarithm also.*

$\ln ab = \ln a + \ln b$

$\ln \frac{a}{b} = \ln a - \ln b$

$2 \ln a = \ln a^2$

## Example 3

Solve each equation using the natural logarithm function.

a) $14^x = 20$

$\ln 14^x = \ln 20$

$\frac{x \cancel{\ln 14}}{\cancel{\ln 14}} = \frac{\ln 20}{\ln 14}$

$x \approx 1.135$

b) $18^{x/2} = 5$

$\ln 18^{x/2} = \ln 5$

$2 \cdot \frac{\frac{x}{2} \ln 18}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$

$x \approx 1.114$

## Example 4

Write an equivalent exponential or logarithmic equation.

a) $e^x = 16$

$\cancel{\ln e}^x = \ln 16$

$x = \ln 16$

b) $e^x = 44$

$\cancel{\ln e}^x = \ln 44$

$x = \ln 44$

c) $\ln 8 \approx 2.079$

$\log_e 8 \approx 2.079$

$e^{2.079} \approx 8$

d) $\ln \frac{1}{4} \approx -1.386$

$\log_e \frac{1}{4} \approx -1.386$

$e^{-1.386} \approx \frac{1}{4}$

Most of the carbon found in the Earth's atmosphere is the isotope carbon-12, but a small amount is the radioactive isotope carbon-14. Plants absorb carbon dioxide from the atmosphere, and animals obtain carbon from the plants they consume. When a plant or animal dies, the amount of carbon-14 it contains decays in such a way that exactly half of its initial amount is present after 5730 years. The function below models the decay of carbon-14, where $N_0$ is the initial amount of carbon-14 and N(t) is the amount present t years after the plant or animal dies.

$$\boxed{N(t) = N_0 e^{-0.00012t}}$$

## Example 5

Suppose that archeologists find scrolls and claim that they are 2000 years old. Tests indicate that the scrolls contain 78% of their original carbon-14. Could the scrolls be 2000 years old?

$$N(t) = N_0 e^{-0.00012t}$$

$$\frac{.78\cancel{N_0}}{\cancel{N_0}} = \frac{\cancel{N_0} e^{-0.00012t}}{\cancel{N_0}}$$

$$.78 = e^{-0.00012t}$$

$$\ln .78 = \cancel{\ln e}^{-0.00012t}$$

$$\frac{\ln .78}{-0.00012} = \frac{\cancel{-0.00012}t}{\cancel{-0.00012}}$$

$$-2070.5 \approx t$$

years

yes

## Example 6

Suppose that a jar containing grain is found at an archeological dig and that archeologists claim that it is 6500 years old. Tests indicate that the grain contains 62% of its original carbon-14. Could the grain be 6500 years old?

$$N(t) = N_0 e^{-0.00012t}$$

$$\frac{.62\cancel{N_0}}{\cancel{N_0}} = \frac{\cancel{N_0} e^{-0.00012t}}{\cancel{N_0}}$$

$$.62 = e^{-0.00012t}$$

$$\ln .62 = \cancel{\ln e}^{-0.00012t}$$

$$\frac{\ln .62}{-0.00012} = \frac{-\cancel{0.00012}t}{\cancel{-0.00012}}$$

$$3983.6 \approx t$$

No

## Example 7

The factory sales of pagers from 1990 through 1995 can be modeled by the function $S = 116e^{0.18t}$, where t = 0 in 1990 and S represents the sales in millions of dollars.

a) Find the factory sales of pagers in $\underset{}{\overset{t=5}{1995}}$ to the nearest million.

b) If the sales of pagers continued to increase at the same rate, when would the sales be double the 1995 amount?

a) $S = 116e^{0.18(5)} \approx 285$ million dollars

b) Find t when S = 570.

$$\frac{570}{116} = \frac{116e^{0.18t}}{116}$$

$$\ln \frac{570}{116} = \cancel{\ln e}^{0.18t}$$

$$\frac{\ln \frac{570}{116}}{.18} = \frac{0.18t}{.18}$$

$$t \approx 8.8 \rightarrow \boxed{\text{end of } 1998}$$