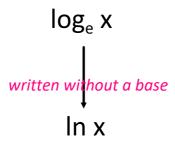
6.6 Part 2 The Natural Logarithm

Recall that we learned about e^x , the natural exponential function.

The natural logarithm is a logarithm with a base of e.



Example 1

Evaluate each using a calculator.

$$2.079$$
 -3.219 undefined 4.605

Recall that logarithms are inverses of exponential functions.

So the inverse of e^x is $\ln x$.

This means $\ln e^{x} = \underline{x}$ and $e^{\ln x} = \underline{x}$.

Example 2

Simplify each expression.

a)
$$e^{\ln 9}$$
 b) $\frac{\ln e^{x/2}}{2}$ c) $e^{\ln 20}$

The properties of logarithms apply to the natural logarithm also.

$$\ln ab = \ln a + \ln b$$

$$\ln \frac{a}{b} = \ln a - \ln b$$

$$2 \ln a^{2} = \ln a^{2}$$

Example 3

Solve each equation using the natural logarithm function.

Solve each equation using the natural logarithm

a)
$$14^{x} = 20$$

b) $18^{x/2} = 5$
 $\ln 14^{x} = \ln 20$
 $1 \ln 18^{x} = \ln 5$
 $2 \cdot \frac{x}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$
 $2 \cdot \frac{x}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$
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b)
$$18^{x/2} = 5$$
In $18^{x/2} = \ln 5$

$$2 \cdot \frac{2}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$$

$$2 \cdot \frac{2}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$$

$$2 \cdot \frac{2}{\ln 18} = \frac{\ln 5}{\ln 18} \cdot 2$$

Example 4

Write an equivalent exponential or logarithmic equation.

a)
$$e^x = 16$$
 $ln e^x = ln 16$
 $x = ln 16$

)
$$e^{x} = 16$$

 $\ln e^{x} = \ln 16$
 $= \ln 16$
b) $e^{x} = 44$
 $\ln e^{x} = \ln 44$
 $= \ln 16$
b) $e^{x} = 44$
 $= \ln 44$
 $= \ln 44$

c)
$$\ln 8 \approx 2.079$$
 $\log_e 8 \approx 2.079$
 $e^{2.079} \approx 8$

d)
$$\ln \frac{1}{4} \approx -1.386$$

 $\log_e \frac{1}{4} \approx -1.386$
 $e^{-1.386} \approx \frac{1}{4}$

Most of the carbon found in the Earth's atmosphere is the isotope carbon-12, but a small amount is the radioactive isotope carbon-14. Plants absorb carbon dioxide from the atmosphere, and animals obtain carbon from the plants they consume. When a plant or animal dies, the amount of carbon-14 it contains decays in such a way that exactly half of its initial amount is present after 5730 years. The function below models the decay of carbon-14, where N₀ is the initial amount of carbon-14 and N(t) is the amount present t years after the plant or animal dies.

$$N(t) = N_0 e^{-0.00012t}$$

Example 5

Suppose that archeologists find scrolls and claim that they are 2000 years old. Tests indicate that the scrolls contain 78% of their original carbon-14. Could the scrolls be 2000 years old?

N(+) = N $e^{-0.000!2+}$

$$N(t) = N_0 e^{-0.000!2t}$$

$$.78 M_0 = M_0 e^{-0.00012t}$$

$$.78 = e^{-0.00012t}$$

$$In .78 = In e^{-0.00012t}$$

$$In .78 = -0.00012t$$

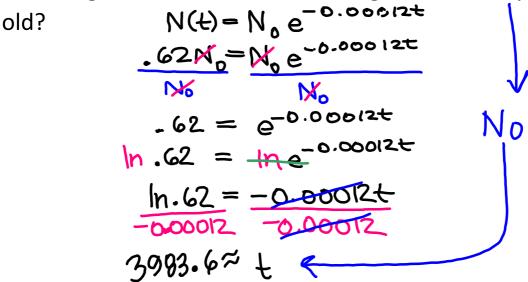
$$-0.00012$$

$$-0.00012$$

$$-0.00012$$

Example 6

Suppose that a jar containing grain is found at an archeological dig and that archeologists claim that it is 6500 years old. Tests indicate that the grain contains 62% of its original carbon-14. Could the grain be 6500 years



Example 7

The factory sales of pagers from 1990 through 1995 can be modeled by the function $S = 116e^{0.18t}$, where t = 0 in 1990 and S represents the sales in millions of dollars.

- a) Find the factory sales of pagers in 1995 to the nearest million.
- b) If the sales of pagers continued to increase at the same rate, when would the sales be double the 1995 amount?

a)
$$S = 116e^{0.18(5)} \approx 285$$
 million dollars
b) Find t when $S = 570$.

$$\frac{570}{116} = \frac{116e^{0.18t}}{116}$$

$$\ln \frac{570}{116} = \ln e^{0.18t}$$

$$\ln \frac{570}{116} = \frac{0.18t}{.18}$$

$$+ \approx 8.8 \rightarrow \frac{1908}{1908}$$