5.3 Trigonometric Graphs

Remember that sin and cos
repeat their values in a regular fashion.

The sin and cos functions are <u>periodic</u>.
The period of sin and cos is 2π (which means they
repeat their values every 2π).

Therefore, to sketch their graphs,
we need to first graph one period.

Example 1

Graph $y = \cos t$.

We will make a table for the first time we graph cos t.

t	y
0	1
π/6	.866
π/4	.707
π/3	.5
π/2	0
2π/3	−.5
3π/4	−.707
5π/6	−.866
π	−1
7π/6	−.866
5π/4	−.707
4π/3	−.5
3π/2	0
5π/3	.5
7π/4	.707
11π/6	.866
2π	1

valley
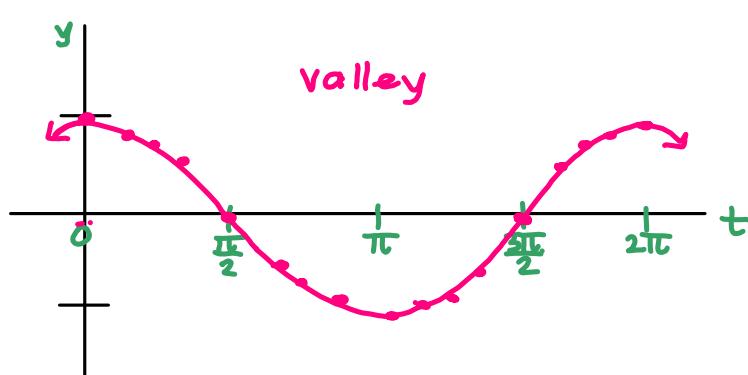
*The graph of y = cos t
is symmetric with
respect to what?*

y-axis

Graphs of Transformations of sin and cos

(we are now going to use *x* instead of *t*)

$$y = a \sin k(x - b) + v \quad \text{or} \quad y = a \cos k(x - b) + v$$

$a \Rightarrow$ amplitude ($|a|$ is the distance between the *x*-axis and the highest/lowest point on the graph

$k \Rightarrow$ horizontal stretch or compression (changes the period)
$\Rightarrow$ to find the new period divide 2π by *k*

$b \Rightarrow$ phase shift (previously referred to as a horizontal shift)

$v \Rightarrow$ vertical shift

Example 2
Graph $f(x) = -4 + \cos x.$ $f(x) = \cos \overset{k=1}{x} - 4$

amplitude = $\underline{1}$

period = 2π

phase shift = $\varnothing$

vertical shift = 4 down
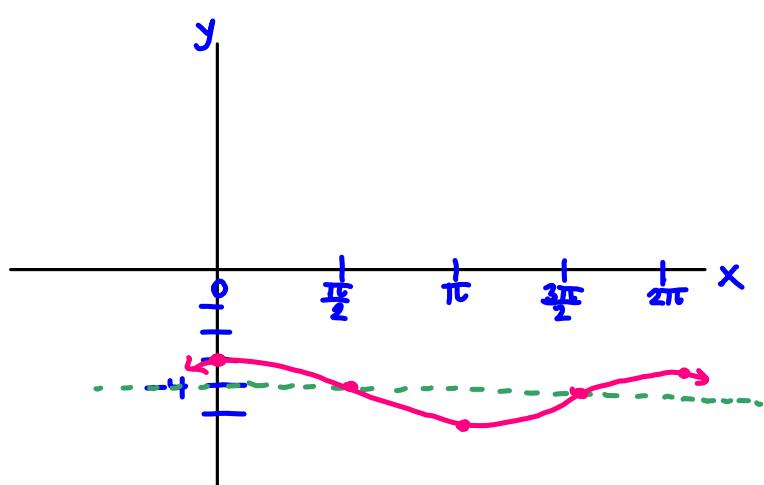
Example 3

Graph $f(x) = 5 \cos x$.

amplitude = 5

period = 2π

phase shift = $\emptyset$
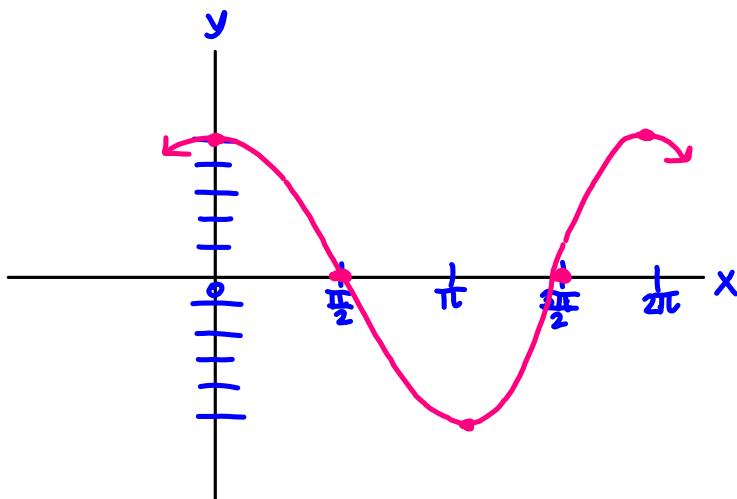
vertical shift = $\emptyset$

Example 4

Graph $f(x) = \cos 2x$.

amplitude = 1

period = $\dfrac{2\pi}{2} = \pi$

phase shift = $\emptyset$

vertical shift = $\emptyset$
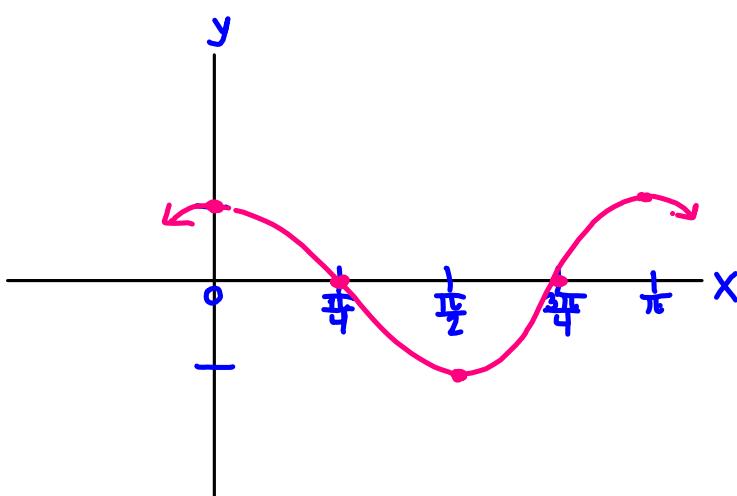
Example 5

Graph $f(x) = 1 + \frac{1}{2} \cos \frac{\pi}{4} x$.

amplitude = $\frac{1}{2}$

period = $\frac{2\pi}{\frac{\pi}{4}} = 2\pi \cdot \frac{4}{\pi} = 8$

phase shift = $\emptyset$

vertical shift = up 1
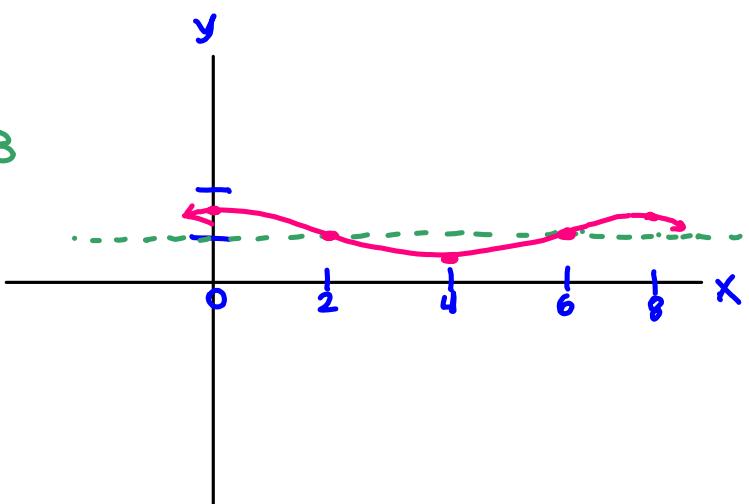
Example 6

Graph $f(x) = \cos\left(x + \frac{\pi}{2}\right)$.

amplitude = 1

period = 2π

phase shift = $\frac{\pi}{2}$ left

vertical shift = $\emptyset$

$-\frac{\pi}{2} + 2\pi = -\frac{\pi}{2} + \frac{4\pi}{2} = \frac{3\pi}{2}$

$\frac{1}{2} \cdot \left(-\frac{\pi}{2} + \frac{3\pi}{2}\right) = \frac{1}{2} \cdot \frac{2\pi}{2} = \frac{\pi}{2}$

$\frac{1}{2}\left(\frac{\pi}{2} + \frac{3\pi}{2}\right) = \frac{1}{2}\left(\frac{4\pi}{2}\right) = \pi$
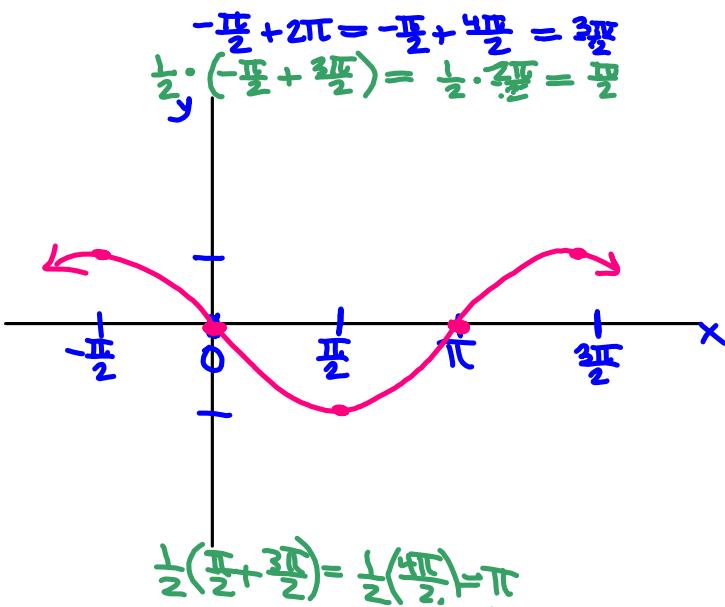
Example 7

Graph $f(x) = 2 \circlearrowleft \frac{1}{2} \cos \pi (x + 1).$

　　↓
　reflection

amplitude = $\frac{1}{2}$

period = $\frac{2\pi}{\pi} = 2$

phase shift = left 1

vertical shift = 2 up
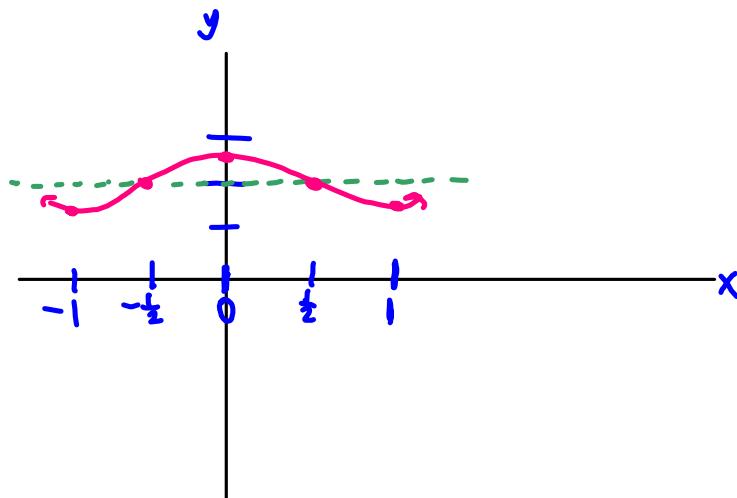
Example 8

$\frac{3}{2} \cos 2 \left(x - \frac{\pi}{6} \right) + \frac{1}{2}$

Graph $f(x) = \frac{1}{2} + \frac{3}{2} \cos \left(2x - \frac{\pi}{3} \right).$

amplitude = $\frac{3}{2}$

period = $\frac{2\pi}{2} = \pi$

phase shift = $\frac{\pi}{6}$ right

vertical shift = $\frac{1}{2}$ up

$\frac{\pi}{6} + \frac{4\pi}{6} = \frac{2\pi}{6}$

$\frac{1}{2}\left(\frac{\pi}{6} + \frac{4\pi}{6}\right) = \frac{1}{2}\left(\frac{3\pi}{6}\right) = \frac{3\pi}{12} = \frac{2\pi}{3}$

$\frac{1}{2}\left(\frac{\pi}{6} + \frac{4\pi}{3}\right) = \frac{1}{2}\left(\frac{\pi}{6} + \frac{4\pi}{6}\right) = \frac{1}{2}\left(\frac{5\pi}{6}\right) = \frac{5\pi}{12}$

$\frac{1}{2}\left(\frac{2\pi}{3} + \frac{7\pi}{6}\right) = \frac{1}{2}\left(\frac{4\pi}{6} + \frac{7\pi}{6}\right) = \frac{1}{2}\left(\frac{11\pi}{6}\right) = \frac{11\pi}{12}$