5.3 Trigonometric Graphs

Remember that sin and cos
repeat their values in a regular fashion.

The sin and cos functions are periodic.
The period of sin and cos is 2π (which means they repeat their values every 2π).

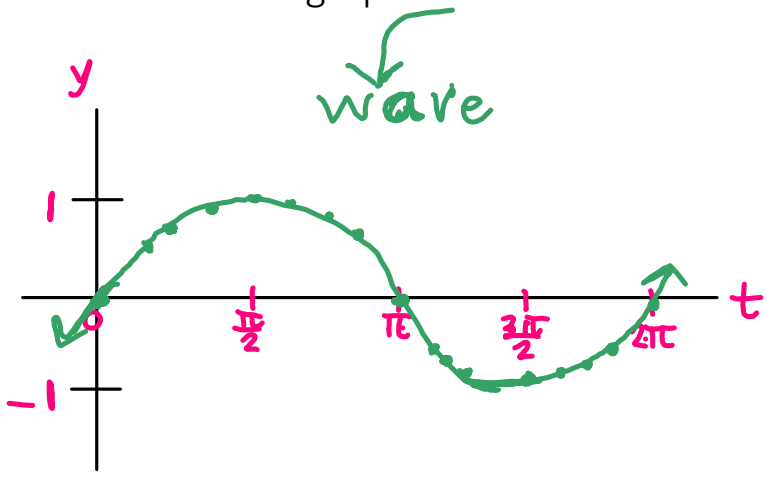
Therefore, to sketch their graphs,
we need to first graph one period.

Example 1

Graph $y = \sin t$.

We will make a table for the first time we graph sin *t*.

t	*y*
0	0
π/6	.5
π/4	.707
π/3	.866
π/2	1
2π/3	.866
3π/4	.707
5π/6	.5
π	0
7π/6	−.5
5π/4	−.707
4π/3	−.866
3π/2	−1
5π/3	−.866
7π/4	−.707
11π/6	−.5
2π	0



The graph of y = sin t is symmetric with respect to what?

origin

Graphs of Transformations of sin and cos

(we are now going to use x instead of t)

$y = a \sin k(x - b) + v$ or $y = a \cos k(x - b) + v$

$a \Rightarrow$ amplitude ($|a|$ is the distance between the x-axis and the highest/lowest point on the graph

$k \Rightarrow$ horizontal stretch or compression (changes the period)
$\Rightarrow$ to find the new period divide 2π by k

$b \Rightarrow$ phase shift (previously referred to as a horizontal shift)

$v \Rightarrow$ vertical shift

Example 2

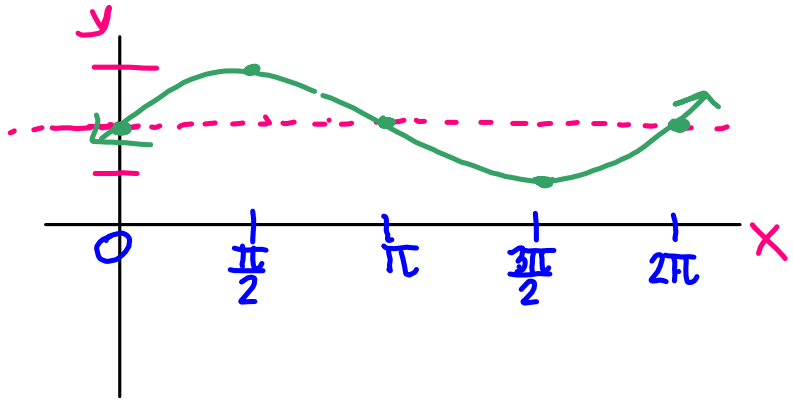
Graph $f(x) = 2 + \sin x$. $k = 1$ $= \sin x + 2$

amplitude = 1

period = $\frac{2\pi}{1} = 2\pi$

phase shift = $\emptyset$ start at 0

vertical shift = up 2



Example 3

$k = 1$

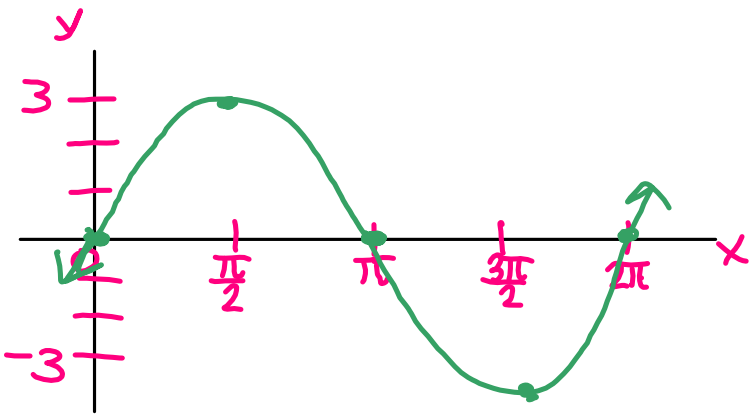
Graph $f(x) = 3 \sin x$.

amplitude = 3

period = $\frac{2\pi}{1} = 2\pi$

phase shift = $\emptyset$

vertical shift = $\emptyset$



Example 4

reflection over x-axis

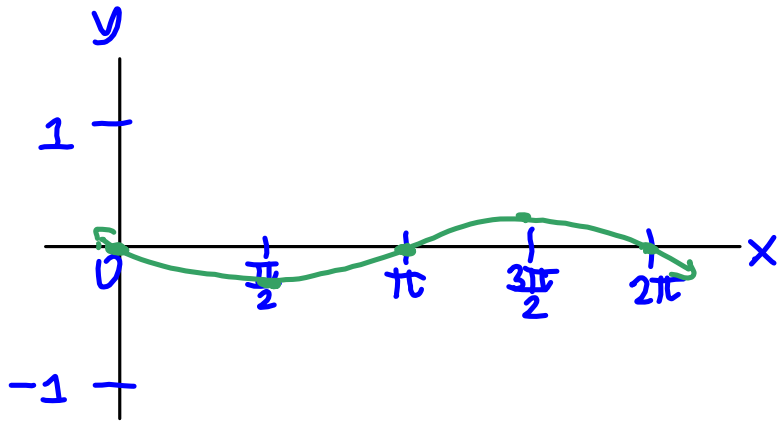
Graph $f(x) = -\frac{1}{4} \sin x$.

amplitude = $\frac{1}{4}$

period = 2π

phase shift = $\emptyset$

vertical shift = $\emptyset$



Example 5

Graph $f(x) = 2 \sin \frac{1}{2}x$.

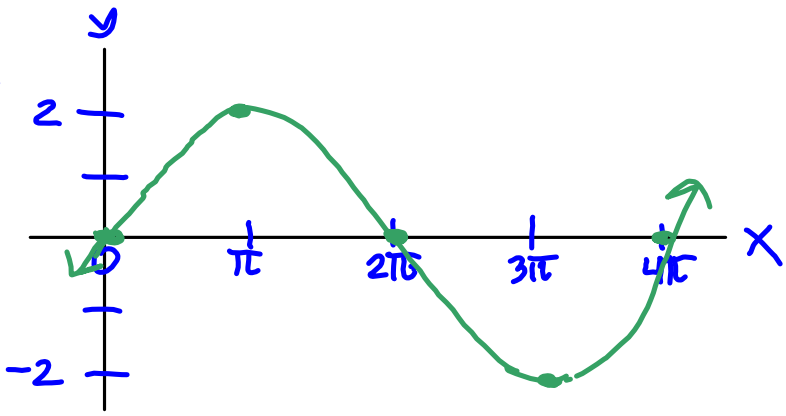
k → how much of the wave you'll see in a normal 2π period

amplitude = 2

period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

phase shift = $\emptyset$

vertical shift = $\emptyset$



Example 6

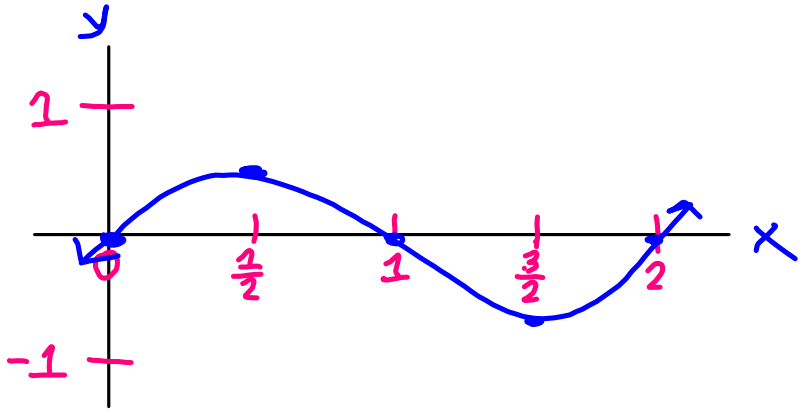
Graph $f(x) = \frac{1}{2} \sin \pi x$.

amplitude = $\frac{1}{2}$

period = $\frac{2\pi}{\pi} = 2$

phase shift = $\emptyset$

vertical shift = $\emptyset$



Example 7

Graph $f(x) = 3 \sin 2(x - \frac{\pi}{4})$.

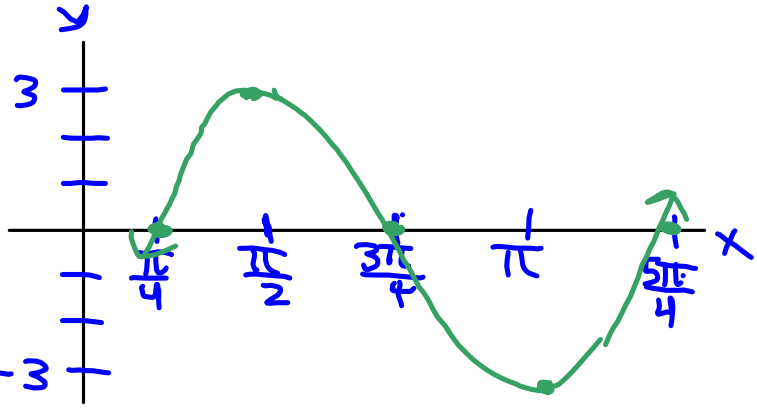
$\frac{1}{2} \cdot (\frac{\pi}{4} + \frac{5\pi}{4}) = \frac{1}{2} \cdot \frac{6\pi}{4} = \frac{3\pi}{4}$

amplitude = 3

period = $\frac{2\pi}{2} = \pi$

phase shift = $\frac{\pi}{4}$ right

vertical shift = $\emptyset$



start $\frac{\pi}{4} + \pi = \frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4}$ end

Example 8

$\frac{3}{2} \sin 2(x - \frac{2\pi}{3}) + 2$

Graph $f(x) = \frac{3}{2} \sin (2x - \frac{4\pi}{3}) + 2$.

$\frac{1}{2}(\frac{4\pi}{6} + \frac{7\pi}{6}) = \frac{1}{2} \cdot \frac{11\pi}{6} = \frac{11\pi}{12}$

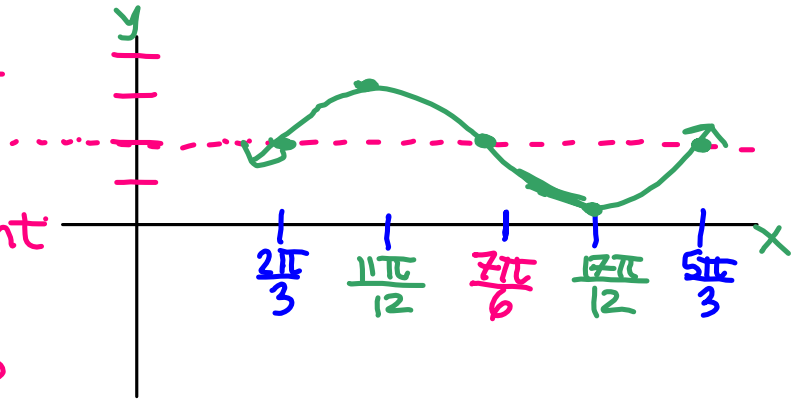
$\frac{1}{2}(\frac{7\pi}{6} + \frac{10\pi}{6}) = \frac{1}{2} \cdot \frac{17\pi}{6} = \frac{17\pi}{12}$

amplitude = $\frac{3}{2}$

period = $\frac{2\pi}{2} = \pi$

phase shift = $\frac{2\pi}{3}$ right

vertical shift = 2 up



$\frac{2\pi}{3} + \pi = \frac{2\pi}{3} + \frac{3\pi}{3} = \frac{5\pi}{3}$ end

$\frac{1}{2} \cdot (\frac{2\pi}{3} + \frac{5\pi}{3}) = \frac{1}{2} \cdot \frac{7\pi}{3} = \frac{7\pi}{6}$