

5.2 Part 1

Trigonometric Functions of Real Numbers

There are 6 **trigonometric functions** that are defined using the x - and y -coordinates of the terminal point.

sine

$$\sin t = y$$

cosecant

$$\csc t = \frac{1}{y} \quad (y \neq 0)$$

cosine

$$\cos t = x$$

secant

$$\sec t = \frac{1}{x} \quad (x \neq 0)$$

tangent

$$\tan t = \frac{y}{x} \quad (x \neq 0)$$

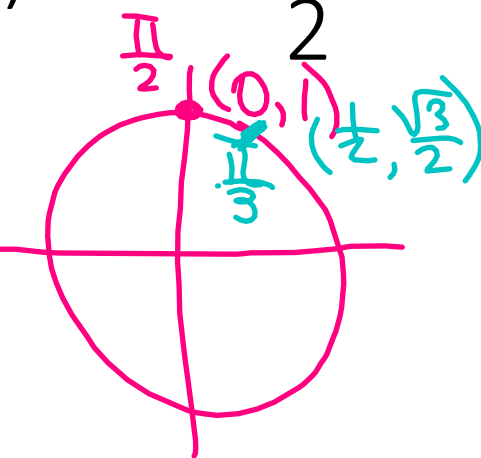
cotangent

$$\cot t = \frac{x}{y} \quad (y \neq 0)$$

Example 1

Find the 6 trigonometric functions of each real number t . (Hint: Find the terminal point.)

a) $\frac{\pi}{2}t =$



$\sin \frac{\pi}{2} = 1$
 $\cos \frac{\pi}{2} = 0$
 $\tan \frac{\pi}{2} = \text{undefined}$

$$\csc \frac{\pi}{2} = 1$$

$$\sec \frac{\pi}{2} = \text{undefined}$$

$$\cot \frac{\pi}{2} = 0$$

b) $\frac{\pi}{3}t =$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos \frac{\pi}{3} = \frac{1}{2}$$
$$\tan \frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec \frac{\pi}{3} = 2$$

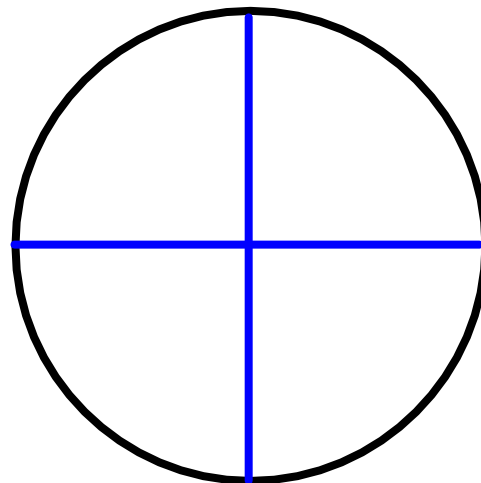
$$\cot \frac{\pi}{3} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Domains of the Trigonometric Functions

sin & **cos**: all real numbers

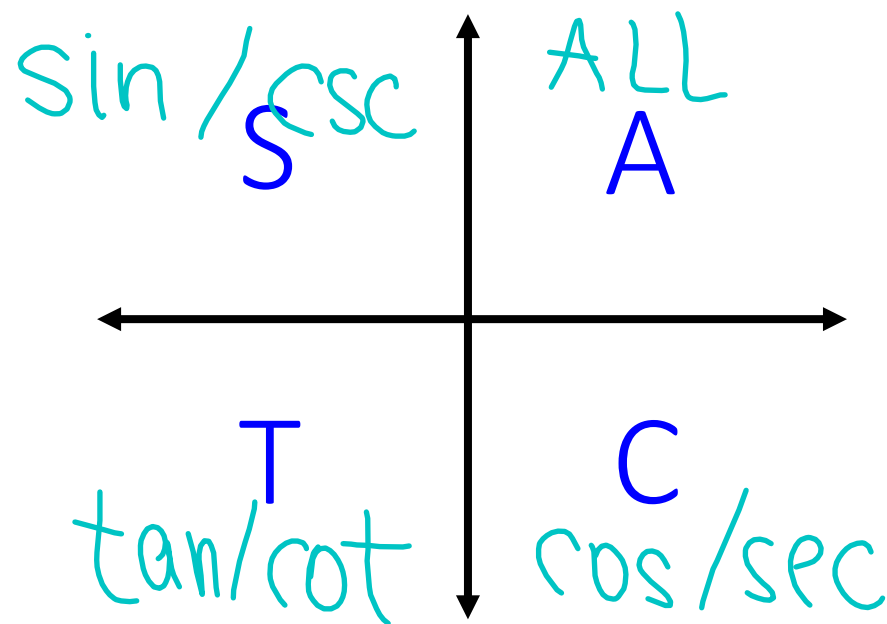
tan & **sec**: all real numbers other than

cot & **csc**: all real numbers other than $\frac{\pi}{2} \pm n\pi$ for any integer n
 $n\pi$ for any integer n



Recall that the terminal points
(and therefore the trig function values) are the
same numerical value with different signs.

Positive Trig Functions



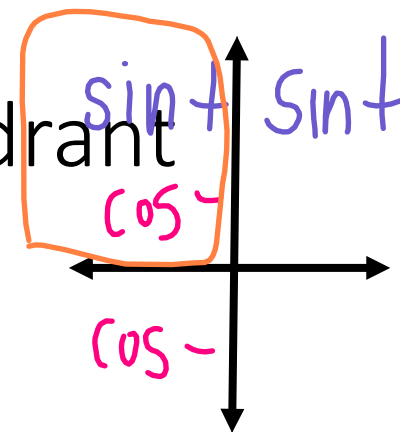
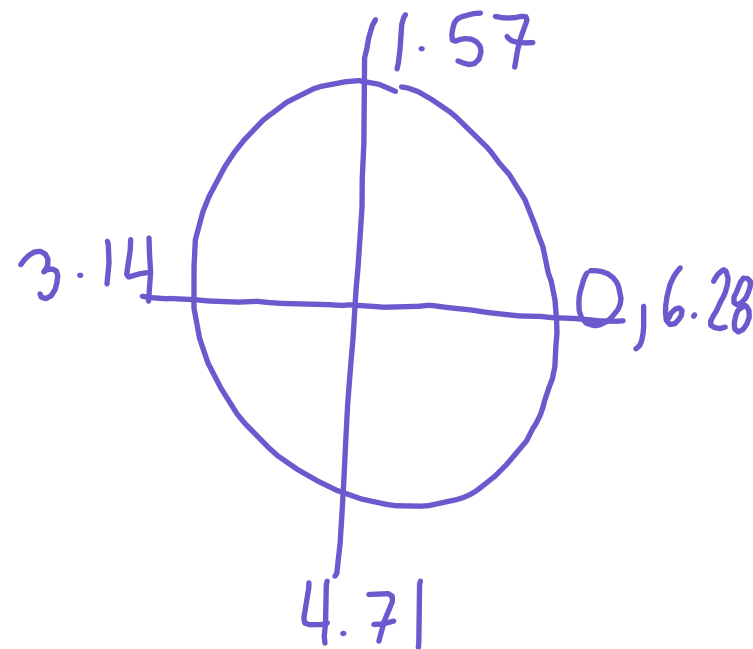
Example 2

Determine the sign of the following functions.

a) $\cos \frac{\pi}{3}$ positive
QI

b) $\tan 6$ negative
QIV

c) If $\cos t < 0$ and $\sin t > 0$, in what quadrant must the terminal point lie?



Example 3

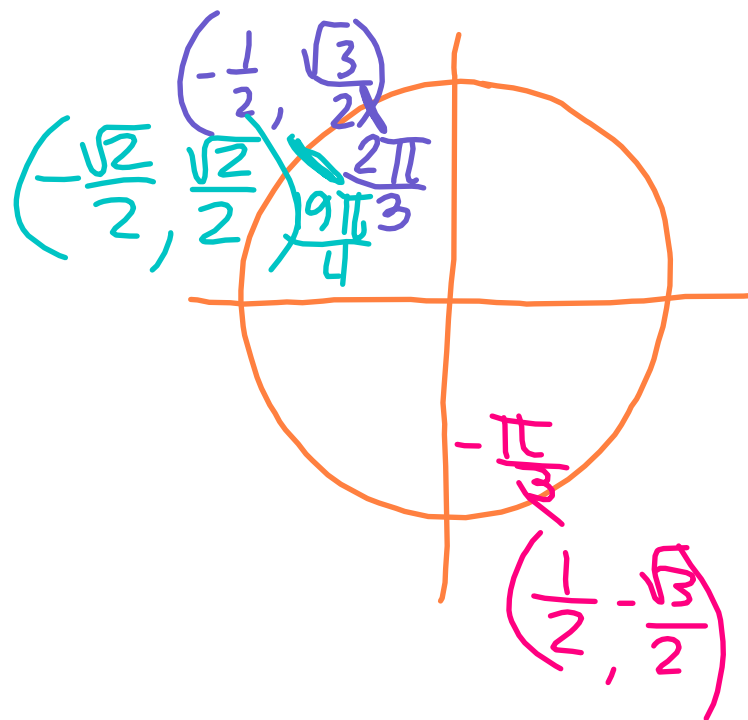
Find each value.

$$\text{a) } \cos \frac{2\pi}{3} = -\frac{1}{2}$$

$$\text{b) } \tan \left(-\frac{\pi}{3} \right) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

$$\text{c) } \sin \frac{19\pi}{4} = \frac{\sqrt{2}}{2}$$

$4\pi + \frac{3\pi}{4}$



Example 4

The terminal point $P(x, y)$ determined by t is given. Find $\sin t$, $\cos t$, $\tan t$, $\csc t$, $\sec t$, & $\cot t$.

a) $\left(-\frac{3}{5}, \frac{4}{5}\right)$

$$\sin t = \frac{4}{5} \qquad \csc t = \frac{5}{4}$$
$$\cos t = -\frac{3}{5} \qquad \sec t = -\frac{5}{3}$$
$$\tan t = \frac{\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{5} \cdot -\frac{5}{3} = -\frac{4}{3} \qquad \cot t = -\frac{3}{4}$$

b) $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$

$$\sin t = \frac{-2\sqrt{2}}{3} \qquad \csc t = \frac{-3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$
$$\cos t = -\frac{1}{3} \qquad \sec t = -3$$
$$\tan t = \frac{-2\sqrt{2}}{3} \cdot -\frac{3}{1} = 2\sqrt{2} \qquad \cot t = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$