6.3 Operations with Polynomials

monomial - a number, a variable, or a product of numbers and variables $-4,\ y,\ \frac{2}{3}a^2b$

constant - a monomial with no variables 7

coefficient - the numerical factor in a monomial $6x$ (coefficient: 6)

degree of a monomial - the sum of the exponents of its variables

$4y^3$ deg $= 3$ $\qquad \frac{1}{2}a^2b^9$ deg $= 2+9 = 11$

polynomial - a monomial or a sum of terms that are monomials

binomial - a polynomial with two terms

trinomial - a polynomial with three terms

degree of a polynomial - the same as that of its term with the greatest degree

$7d^3+8d^2-9d+1$: d^3 deg $= 3$, d^2 deg $= 2$, d deg $= 1$, 1 deg $= 0$

$8xy^2-5x^3y+10x^2y^5$: deg $= 1+2 = 3$, deg $= 3+1 = 4$, deg $= 2+5 = 7$

CLASSIFICATION OF POLYNOMIALS BY DEGREE

DEGREE	NAME	EXAMPLE
0	constant	-29
1	linear	$2x-5$
2	quadratic	$-3x^2+2x-8$
3	cubic	$4x^3-10x$
4	quartic	$-5x^4+x^3-2x+9$
5	quintic	x^5-8x^2+12

EXAMPLE: Rewrite each polynomial in descending order (highest → lowest). Then classify each polynomial by its degree and number of terms.

1. $2x^2 - 8x - 4x^4$

$-4x^4 + 2x^2 - 8x$

quartic trinomial

2. $-9x + 10x^3$

$10x^3 - 9x$

cubic binomial

3. $-x^4 + 9x + 7x^5 - 6$

$7x^5 - x^4 + 9x - 6$

quintic polynomial

To add and subtract polynomials, combine like terms. Write your answer in standard form, which is with the exponents in ***descending*** order of degree.

4. Find the sum.

$(-2x^2 - 3x^3 + 5x + 4) + (-2x^3 + 7x - 6)$

$-5x^3 - 2x^2 + 12x - 2$

5. Find the difference.

$(-6x^3 - 6x^2 + 7x - 1) + (3x^3 + 5x^2 + 2x + 8)$

$-9x^3 - 1x^2 + 9x - 9$

6. $(2x^4 + 4x^3 + 5x - 2) + (-2x^4 - 7x^2 + 8x - 10)$

$4x^3 - 7x^2 + 13x - 12$

7. $(3x^3 - 12x^2 - 5x + 1) + (+x^2 + 5x + 8)$

$3x^3 - 11x^2 - 10x - 7$

8. Multiply $(x - 2)(5x^2 + 3x - 4)$.

$x(5x^2 + 3x - 4) \quad -2(5x^2 + 3x - 4)$

$5x^3 + 3x^2 - 4x \quad -10x^2 - 6x + 8$

$5x^3 - 7x^2 - 10x + 8$

9. Multiply $(x - 1)(x + 2)(x - 3)$.

FOIL

$(x-1)(x^2-3x+2x-6)$

$(x-1)(x^2-1x-6)$

$x(x^2-1x-6) \quad -1(x^2-1x-6)$

$x^3-x^2-6x \quad -x^2+x+6$

x^3-2x^2-5x+6

10. Multiply $(x + 4)^3$.

FOIL

$(x+4)(x+4)(x+4)$

$(x+4)(x^2+4x+4x+16)$

$(x+4)(x^2+8x+16)$

$x(x^2+8x+16) \quad +4(x^2+8x+16)$

$x^3+8x^2+16x \quad +4x^2+32x+64$

$x^3+12x^2+48x+64$

Cube of a Binomial

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

11. Multiply $(\boxed{4b}\ -\boxed{7})^3$. ($a = 4b$, $b = 7$)

$a^3 - 3a^2b + 3ab^2 - b^3$

$(4b)^3 - 3(4b)^2(7) + 3(4b)(7)^2 - (7)^3$

$64b^3 - 3(16b^2)(7) + 3(4b)(49) - 343$

$64b^3 - 336b^2 + 588b - 343$

12. Multiply $(\boxed{5y}+\boxed{3})^3$. ($a = 5y$, $b = 3$)

$a^3 + 3a^2b + 3ab^2 + b^3$

$(5y)^3 + 3(5y)^2(3) + 3(5y)(3)^2 + (3)^3$

$125y^3 + 3(25y^2)(3) + 3(5y)(9) + 27$

$125y^3 + 225y^2 + 135y + 27$