6.5 Applications of Common Logarithms

Solving Equations Without a Calculator

Sometimes we can solve exponential equations without a calculator simply by changing the bases.

Example 1: Solve $4^{x} = \left(\frac{1}{2}\right)^{x - 3}$.

$(2^{2})^{x} = (2^{-1})^{x-3}$

$2x = -1(x-3)$

$2x = -x + 3$

$+x \qquad +x$

$3x = 3$

$\frac{3x}{3} = \frac{3}{3}$

$\boxed{x=1}$

Example 2: Solve $9^{2x} = 27^{x - 1}$.

$(3^{2})^{2x} = (3^{3})^{x-1}$

$2(2x) = 3(x-1)$

$4x = 3x - 3$

$-3x \qquad -3x$

$\boxed{x = -3}$

Example 3: Solve $100^{7x+1} = 1000^{3x-2}$.

$$(10^{2})^{7x+1} = (10^{3})^{3x-2}$$

$$2(7x+1) = 3(3x-2)$$

$$14x + 2 = \cancel{9x} - 6$$

$$-9x \qquad \cancel{-9x}$$

$$5x + \cancel{2} = -6$$

$$\cancel{-2} \qquad -2$$

$$\frac{5x}{5} = \frac{-8}{5}$$

$$\boxed{x = -\frac{8}{5}}$$

Example 4: Solve $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$.

$$(3^{4})^{3-x} = (3^{-1})^{5x-6}$$

$$4(3-x) = -1(5x-6)$$

$$12 - 4x = -\cancel{5x} + 6$$

$$+5x \qquad \cancel{+5x}$$

$$12 + x = 6$$

$$-12 \qquad -12$$

$$\boxed{x = -6}$$

Solving Equations With a Calculator

If we cannot rewrite each side to have a common base, we must use a logarithm and a calculator.

Example 5: Solve $4^x = 11$.

$\log 4^x = \log 11$

$\frac{x \cdot \cancel{\log 4}}{\cancel{\log 4}} = \frac{\log 11}{\log 4}$

$\boxed{x \approx 1.730}$

Example 6: Solve $15^{-x} = 7$.

$\log 15^{-x} = \log 7$

$\frac{-x \cancel{\log 15}}{\cancel{-\log 15}} = \frac{\log 7}{-\log 15}$

$\boxed{x \approx -0.719}$

Example 7: Solve $4^{-x} + 2 = 11$.

$$\underline{\quad -2 \quad -2}$$

$$4^{-x} = 9$$

$$\log 4^{-x} = \log 9$$

$$\frac{-x \log 4}{-\log 4} = \frac{\log 9}{-\log 4}$$

$$\boxed{x \approx -1.585}$$

Example 8: Solve $1 - \left(\frac{2}{3}\right)^x = -6$.

$$\underline{-1 \qquad -1}$$

$$\frac{-\left(\frac{2}{3}\right)^x}{-1} = \frac{-7}{-1}$$

$$\left(\frac{2}{3}\right)^x = 7$$

$$\log\left(\frac{2}{3}\right)^x = \log 7$$

$$\frac{x \cancel{\log \frac{2}{3}}}{\cancel{\log \frac{2}{3}}} = \frac{\log 7}{\log \frac{2}{3}}$$

$$\boxed{x \approx -4.799}$$

The calculator only uses a base of 10 for logarithms. So when we have a logarithm with any other base, we need another way to evaluate it.

CHANGE OF BASE FORMULA

For any positive real numbers $a \neq 1$, $b \neq 1$, and $x > 0$:

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

pick base
(10 every time)

Example 9: Evaluate $\log_7 32$.

$$\frac{\log_{10} 32}{\log_{10} 7} \rightarrow \frac{\log 32}{\log 7} \approx \boxed{1.781}$$

Example 10: Evaluate $\log_{15} 6$.

$$\frac{\log 6}{\log 15} \approx \boxed{.662}$$

Example 11: Evaluate $\log_{\frac{1}{4}} 20$.

$$\frac{\log 20}{\log \frac{1}{4}} \approx \boxed{-2.161}$$