

6.5 Applications of Common Logarithms

Solving Equations Without a Calculator

Sometimes we can solve exponential equations without a calculator simply by changing the bases.

Example 1: Solve $4^x = \left(\frac{1}{2}\right)^{x-3}$.

$$(2)^{2x} = (2)^{-1(x-3)}$$

$$2x = -1(x-3)$$

$$2x = -x + 3$$

$$+x \quad +x$$

$$\hline 3x = 3$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$x = 1$$

Example 2: Solve $9^{2x} = 27^{x-1}$.

$$(3)^{2(2x)} = (3)^{3(x-1)}$$

$$2(2x) = 3(x-1)$$

$$4x = 3x - 3$$

$$-3x \quad -3x$$

$$\hline x = -3$$

Example 3: Solve $100^{7x+1} = 1000^{3x-2}$.

$$(10^2)^{7x+1} = (10^3)^{3x-2}$$

$$2(7x+1) = 3(3x-2)$$

$$14x + 2 = 9x - 6$$

$$\begin{array}{r} 14x + 2 = 9x - 6 \\ -9x \quad \quad -9x \\ \hline 5x + 2 = -6 \\ \quad \quad -2 \quad \quad -2 \\ \hline 5x = -8 \end{array}$$

$$\frac{5x}{5} = \frac{-8}{5}$$

$$\boxed{x = -\frac{8}{5}}$$

Example 4: Solve $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$.

$$(3^4)^{3-x} = (3^{-1})^{5x-6}$$

$$4(3-x) = -1(5x-6)$$

$$12 - 4x = -5x + 6$$

$$\begin{array}{r} 12 - 4x = -5x + 6 \\ +5x \quad \quad +5x \\ \hline 12 + x = 6 \\ -12 \quad \quad -12 \\ \hline x = -6 \end{array}$$

$$\boxed{x = -6}$$

Solving Equations With a Calculator

If we cannot rewrite each side to have a common base, we must use a logarithm and a calculator.

Example 5: Solve $4^x = 11$.

$$\log 4^x = \log 11$$

$$\frac{x \cdot \log 4}{\log 4} = \frac{\log 11}{\log 4}$$

$$x \approx 1.730$$

Example 6: Solve $15^{-x} = 7$.

$$\log 15^{-x} = \log 7$$

$$\frac{-x \log 15}{-\log 15} = \frac{\log 7}{-\log 15}$$

$$x \approx -0.719$$

Example 7: Solve $4^{-x} + 2 = 11$.

$$\begin{aligned} & \frac{-2}{4^{-x}} = \frac{-2}{9} \\ & 4^{-x} = 9 \\ & \log 4^{-x} = \log 9 \\ & \frac{-x \log 4}{-\log 4} = \frac{\log 9}{-\log 4} \\ & \boxed{x \approx -1.585} \end{aligned}$$

Example 8: Solve $1 - \left(\frac{2}{3}\right)^x = -6$.

$$\begin{aligned} & \frac{-1}{-\left(\frac{2}{3}\right)^x} = \frac{-1}{-7} \\ & \left(\frac{2}{3}\right)^x = 7 \\ & \log \left(\frac{2}{3}\right)^x = \log 7 \\ & \frac{x \log \frac{2}{3}}{\log \frac{2}{3}} = \frac{\log 7}{\log \frac{2}{3}} \\ & \boxed{x \approx -4.799} \end{aligned}$$

The calculator only uses a **base of 10** for logarithms.
So when we have a logarithm with any other base,
we need another way to evaluate it.

CHANGE OF BASE FORMULA

For any positive real numbers $a \neq 1$, $b \neq 1$, and $x > 0$:

The diagram shows the change of base formula: $\log_b x = \frac{\log_a x}{\log_a b}$. The formula is enclosed in a yellow box. A pink arrow points from the base 'b' in the denominator to the base 'a' in the numerator of the fraction. A green arrow points from the base 'a' in the denominator to the base 'a' in the numerator. A blue arrow points from the text 'pick base (10 every time)' to the 'a' in both the numerator and denominator of the fraction.

Example 9: Evaluate $\log_7 32$.

$$\frac{\log_{10} 32}{\log_{10} 7} \rightarrow \frac{\log 32}{\log 7} \approx \boxed{1.781}$$

Example 10: Evaluate $\log_{15} 6$.

$$\frac{\log 6}{\log 15} \approx \boxed{.662}$$

Example 11: Evaluate $\log_{\frac{1}{4}} 20$.

$$\frac{\log 20}{\log \frac{1}{4}} \approx \boxed{-2.161}$$