6.5 Applications of Common Logarithms

Solving Equations Without a Calculator

Sometimes we can solve exponential equations without a calculator simply by changing the bases.

Example 1: Solve
$$4^{x} = \left(\frac{1}{2}\right)^{x-3}$$
.
 $(2^{2})^{x} = \left(2^{-1}\right)^{x-3}$.
 $2x = -1(x-3)$
 $2x = -x+3$
 $+x$
 $3x = 3$
 $3x = 3$
 $x = 1$

Example 2: Solve
$$9^{2x} = 27^{x-1}$$
.

$$(3^{2})^{2x} = (3^{3})^{x-1}$$

$$2(2x) = 3(x-1)$$

$$4x = 3x - 3$$

$$-3x$$

$$x = -3$$

Example 3: Solve
$$100^{7x+1} = 1000^{3x-2}$$
.

$$(10^{3})^{3x-2} = (10^{3})^{3x-2}$$

$$2(7x+1) = 3(3x-2)$$

$$14x + 2 = 9x - 6$$

$$-9x$$

$$5x + 2 = -6$$

$$5x = -8$$

$$5$$

$$x = -8$$

Example 4: Solve
$$81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$$
.

$$(3^{-1})^{3-x} = (3^{-1})^{5x-6}$$

$$4(3-x) = -1(5x-6)$$

$$12-4x = -5x + 6$$

$$+5x + 5x$$

$$12 + x = 6$$

$$-12 - 12$$

$$x = -6$$

Solving Equations With a Calculator

If we cannot rewrite each side to have a common base, we must use a logarithm and a calculator.

Example 5: Solve
$$4^x = 11$$
.

$$\log 4^x = \log 1$$

$$x \cdot \log 4 = \log 1$$

$$\log 4$$

$$|\log 4|$$

$$|\log 4|$$

$$|\cos 4|$$

Example 6: Solve
$$15^{-x} = 7$$
.

 $\log 15^{-x} = \log 7$
 $-x \log 15 = \log 7$
 $-\log 15 = \log 5$
 $\times \sim -0.719$

Example 7: Solve
$$4^{-x} + 2 = 11$$
.

 $-2 - 2$
 $| 10g | 4^{-x} | = 9$
 $-x | 10g | 4^{-x} | = | 10g | 9$
 $-| 10g | 4^{-x} | = | 10g | 9$
 $| 10g | 4^{-x} | = | 10g | 9$
 $| 10g | 4^{-x} | = | 10g | 9$
 $| 10g | 4^{-x} | = | 10g | 9$
 $| 10g | 4^{-x} | = | 10g | 9$

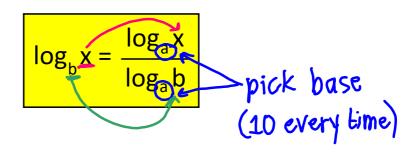
Example 8: Solve
$$1 - \left(\frac{2}{3}\right)^{x} = -6$$
.

 $\frac{-1}{-(\frac{2}{3})^{x}} = -7$
 $\frac{-(\frac{2}{3})^{x}}{-1} = 7$
 $\log\left(\frac{2}{3}\right)^{x} = 7$
 $\log\left(\frac{2}{3}\right)^{x} = \log 7$

The calculator only uses a base of 10 for logarithms. So when we have a logarithm with any other base, we need another way to evaluate it.

CHANGE OF BASE FORMULA

For any positive real numbers a \neq 1, b \neq 1, and x > 0:



Example 9: Evaluate log, 32.

Example 10: Evaluate $\log_{15} 6$.

$$\frac{\log 6}{\log 15} \approx \boxed{.662}$$

Example 11: Evaluate $\log_{\frac{1}{4}} 20$.

$$\frac{\log 20}{\log 4} \approx \boxed{-2.161}$$