

6.4 Properties of Logarithms

Product Property $\log_b m \oplus \log_b \underline{n} = \log_b mn$
num.

Quotient Property $\log_b m \ominus \log_b \underline{n} = \log_b \frac{m}{n}$
denom.

Power Property $n \log_b m = \log_b m^n$

Properties of Logarithms

1. $\log_b 1 = 0$ $b^0 = 1$
2. $\log_b b^1 = 1$ $b^1 = b$
3. $\log_b b^x = x$ $b^x = b^x$
4. $b^{\log_b x} = x$

One-to-One Property of Logarithms

single log on each side
 If $\log_b x = \log_b y$, then $x = y$.
same base

Examples

Given $\log_2 3 \approx 1.5850$, approximate the value of each expression below.

1. $\log_2 12$

$$\begin{aligned} &\log_2 3 \cdot 4 \\ &\log_2 3 + \log_2 4 \\ &1.5850 + \log_2 2^2 = \boxed{3.5850} \end{aligned}$$

2. $\log_2 1.5$

$$\begin{aligned} &\log_2 \frac{3}{2} \\ &\log_2 3 - \log_2 2 \\ &1.5850 - 1 = \boxed{.5850} \end{aligned}$$

Given $\log_3 7 \approx 1.7712$, approximate the value of each expression below.

3. $\log_3 \frac{3}{7}$

$$\begin{aligned} &\log_3 3 - \log_3 7 \\ &1 - 1.7712 \\ &\boxed{-.7712} \end{aligned}$$

4. $\log_3 49$

$$\begin{aligned} &\log_3 7^2 \\ &2 \log_3 7 \\ &2 (1.7712) \\ &\boxed{3.5424} \end{aligned}$$

Examples

Write each expression as a single logarithm. Then simplify, if possible.

condense

5. $\log_2 5 + \log_2 7$

$$\begin{aligned} &\log_2 5 \cdot 7 \\ &\boxed{\log_2 35} \end{aligned}$$

8. $\log_7 3x - \log_7 9x + \log_7 6y$

$$\begin{aligned} &\log_7 \frac{3x \cdot 6y}{9x} \\ &\log_7 \frac{18xy}{9x} = \boxed{\log_7 2y} \end{aligned}$$

6. $\log_3 45 - \log_3 9$

$$\begin{aligned} &\log_3 \frac{45}{9} \\ &\boxed{\log_3 5} \end{aligned}$$

9. $5 \log_2 m - 2 \log_2 n$

$$\begin{aligned} &\log_2 m^5 - \log_2 n^2 \\ &\boxed{\log_2 \frac{m^5}{n^2}} \end{aligned}$$

7. $+\log_2 5 + \log_2 x - \log_2 10$

$$\begin{aligned} &\log_2 \frac{5 \cdot x}{10} \\ &\log_2 \frac{1}{2} x \\ &\text{or} \\ &\log_2 \frac{x}{2} \end{aligned}$$

10. $4 \log_b m + \log_b n - \frac{1}{2} \log_b p$

$$\begin{aligned} &\log_b m^4 + \log_b n - \log_b p^{1/2} \\ &\boxed{\log_b \frac{m^4 n}{p^{1/2}}} \end{aligned}$$

Examples

Write each expression as a ^{expand} sum or difference of logarithms. Then simplify, if possible.

11. $\log_2 4x^3$

$$\begin{aligned} \log_2 4 + \log_2 x^3 \\ \log_2 2^2 + 3\log_2 x \\ 2 + 3\log_2 x \end{aligned}$$

12. $\log_4 \frac{4a}{b^2}$

$$\begin{aligned} \log_4 4 + \log_4 a - \log_4 b^2 \\ 1 + \log_4 a - 2\log_4 b \end{aligned}$$

13. $\log_3 \frac{k^3 m}{9}$

$$\begin{aligned} \log_3 k^3 + \log_3 m - \log_3 9 \\ 3\log_3 k + \log_3 m - \log_3 3^2 \\ 3\log_3 k + \log_3 m - 2 \end{aligned}$$

Examples

Solve and check for extraneous solutions.

14. $\log_3(x^2 + 7x - 5) = \log_3(6x + 1)$

$$\begin{aligned} x^2 + 7x - 5 &= 6x + 1 \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x &= -3 \quad \boxed{x=2} \checkmark \end{aligned}$$

$y = \log_x$

D: $x > 0$
must be positive
 $\log(\#)$

$$\begin{aligned} x = -3 \\ \log_3(9 - 21 - 5) &\stackrel{?}{=} \log_3(-18 + 1) \\ \log_3(-17) &\neq \log_3(-17) \\ x = 2 \\ \log_3(4 + 14 - 5) &\stackrel{?}{=} \log_3(12 + 1) \\ \log_3(13) &= \log_3(13) \checkmark \end{aligned}$$

Examples

Solve and check for extraneous solutions.

condense

15. $2 \log_a x + \log_a 2 = \log_a (5x + 3)$ ✓ $2 \log_a (3) + \log_a (2) \stackrel{?}{=} \log_a (15+3)$

$\log_a x^2 + \log_a 2 = \log_a (5x + 3)$

~~$\log_a 2x^2 = \log_a (5x + 3)$~~

$2x^2 = 5x + 3$

$2x^2 - 5x - 3 = 0$

$(x - 3)(2x + 1) = 0$

✓ $x = 3$ ~~$x = -\frac{1}{2}$~~

$\frac{x=3}{2 \log_a (3) + \log_a (2) \stackrel{?}{=} \log_a (15+3)}$

$\frac{x=-\frac{1}{2}}{2 \log_a (-\frac{1}{2}) \dots \times}$

$\frac{s-5p-6}{\frac{-3-6}{2} \quad \frac{1}{2}}$

Examples

Solve and check for extraneous solutions.

16. $2 \log_3 x + \log_3 5 = \log_3 (14x + 3)$

$\log_3 x^2 + \log_3 5 = \log_3 (14x + 3)$

~~$\log_3 5x^2 = \log_3 (14x + 3)$~~

$5x^2 = 14x + 3$

$5x^2 - 14x - 3 = 0$

$(5x + 1)(x - 3) = 0$

~~$x = -\frac{1}{5}$~~ $x = 3$

$\frac{x=-\frac{1}{5}}{2 \log_3 (-\frac{1}{5}) \dots \times}$

$\frac{x=3}{2 \log_3 (3) + \log_3 (5) = \log_3 (42+3)}$