5.2 Part 2 Trigonometric Functions of Real Numbers

How can we compute the values of the trigonometric functions if t is not a multiple of one of our "special" values? One way is to sketch a diagram and read the value, but this method is not very accurate. The best way to find a close approximation is to use the calculator. The calculator must be in radian mode to evaluate these functions. To find values of cosecant, secant, and cotangent using a calculator, we need to use the reciprocal relations:

$$\csc t = \frac{1}{\sin t} \qquad \sec t = \frac{1}{\cos t} \qquad \cot t = \frac{1}{\tan t}$$

Example 1

Use a calculator to evaluate the following functions to six decimal places.

a) sin 2.2
.808496

b) cos 1.1
.453596

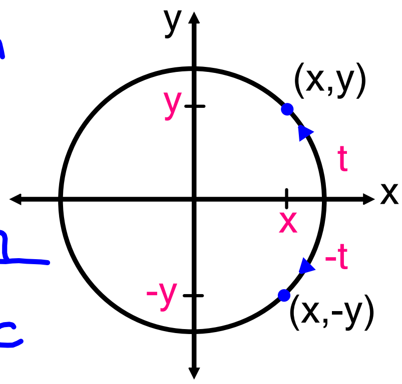
c) cot 28
−3.553286

d) csc 0.98
1.274098

Consider the relationship between the trigonometric functions of t and -t.

odd
sin
tan
csc
cot

even
cos
sec



sin(-t) = $-y$
sin t = y

cos(-t) = x
cos t = x

tan(-t) = $-\frac{y}{x}$
tan t = $\frac{y}{x}$

These equations show that sin and tan are: even or odd functions?
And cos is: even or odd?

The reciprocal of an even function is even
and the reciprocal of an odd function is odd.

Odd functions: sin, tan, csc, cot
Even functions: cos, sec

sin(-t) = $-\sin t$
csc(-t) = $-\csc t$

cos(-t) = $\cos t$
sec(-t) = $\sec t$

tan(-t) = $-\tan t$
cot(-t) = $-\cot t$

Example 2

Determine whether the function is even, odd, or neither.

a) $f(x) = x^2 \cos 2x$

$\cos(-2x)$

$f(-x) = (-x)^2 \cos 2(-x)$

$f(-x) = x^2 \cos 2x$

$\boxed{\text{even}}$

b) $f(x) = e^x \sin x$

$f(-x) = e^{-x} \sin(-x)$

$f(-x) = e^{-x} \cdot -\sin x \rightarrow f(-x) = -e^{-x} \sin x$

$\boxed{\text{neither}}$

c) $f(x) = x \sin^3 x$

$f(-x) = -x[\sin(-x)]^3$

$f(-x) = -x[-\sin x]^3$

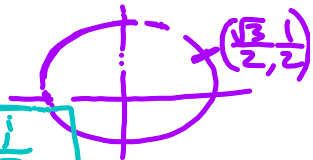
$f(-x) = -x \cdot -\sin^3 x$

$f(-x) = x\sin^3 x$

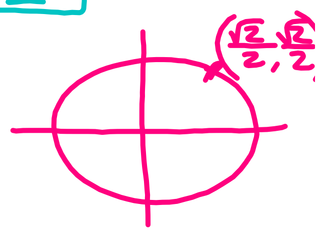
$\boxed{\text{even}}$

Example 3

Use the even-odd properties of the trigonometric functions to determine each value.



a) $\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = \boxed{-\frac{1}{2}}$



b) $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

Fundamental Identities

The trigonometric functions are related to each other through equations called trigonometric identities. The following are the most important.

Reciprocal Identities

$\csc t = \dfrac{1}{\sin t}$ $\sec t = \dfrac{1}{\cos t}$ $\cot t = \dfrac{1}{\tan t}$

$\tan t = \dfrac{\sin t}{\cos t}$ $\cot t = \dfrac{\cos t}{\sin t}$

Pythagorean Identities

1. $\sin^2 t + \cos^2 t = 1$

2. $1 + \cot^2 x = \csc^2 t$

3. $\tan^2 t + 1 = \sec^2 t$

Example 4

S | A
T | C

If cos t = $\frac{3}{5}$ and t is in quadrant IV, find the values of all the trigonometric functions at t.

$\sin t = -\frac{4}{5}$

$\cos t = \frac{3}{5}$

$\tan t = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = -\frac{4}{3}$

$\csc t = -\frac{5}{4}$

$\sec t = \frac{5}{3}$

$\cot t = -\frac{3}{4}$

$\sin^2 t + \cos^2 t = 1$

$\sin^2 t + \left(\frac{3}{5}\right)^2 = 1$

$\sin^2 t + \cancel{\frac{9}{25}} = 1\frac{25}{25}$

$\quad -\cancel{\frac{9}{25}} \quad -\frac{9}{25}$

$\sqrt{\sin^2 t} = \sqrt{\frac{16}{25}}$

$\sin t = \cancel{\pm} - \frac{4}{5}$

Example 5

sin is neg.

S | A
T | C

Write tan t in terms of cos t, where t is in quadrant III.

$\tan t = \frac{\sin t}{\cos t} = \boxed{\frac{-\sqrt{1-\cos^2 t}}{\cos t}}$

Solve for sint.

$\sin^2 t + \cancel{\cos^2 t} = 1$

$\quad -\cancel{\cos^2 t} \quad -\cos^2 t$

$\sqrt{\sin^2 t} = \sqrt{1-\cos^2 t}$

$\sin t = -\sqrt{1-\cos^2 t}$