5.2 Part 1 Trigonometric Functions of Real Numbers

There are 6 trigonometric functions that are defined using the x- and y-coordinates of the terminal point.

sine
$\sin t = y$

cosecant
$\csc t = \frac{1}{y} \quad (y \neq 0)$

cosine
$\cos t = x$

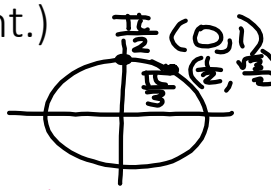
secant
$\sec t = \frac{1}{x} \quad (x \neq 0)$

tangent
$\tan t = \frac{y}{x} \quad (x \neq 0)$

cotangent
$\cot t = \frac{x}{y} \quad (y \neq 0)$

Example 1

Find the 6 trigonometric functions of each real number t. (Hint: Find the terminal point.)



a) $t = \frac{\pi}{2}$

$\sin t = 1$ $\quad \csc t = \frac{1}{1} = 1$

$\cos t = 0$ $\quad \sec t = \frac{1}{0} = \text{undefined}$

$\tan t = \frac{1}{0} = \text{undefined}$ $\quad \cot t = \frac{0}{1} = 0$

b) $t = \frac{\pi}{3}$

$\sin t = \frac{\sqrt{3}}{2}$ $\quad \csc t = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos t = \frac{1}{2}$ $\quad \sec t = 2$

$\tan t = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$ $\quad \cot t = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Domains of the Trigonometric Functions

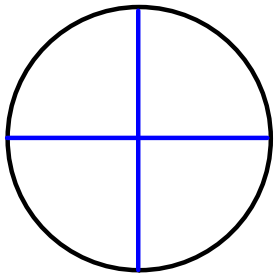
sin & cos: all real numbers

tan & sec: all real numbers other than $\frac{\pi}{2} \pm n\pi$ for any integer n

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

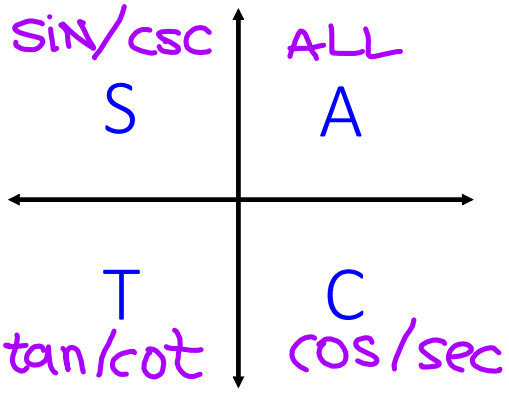
cot & csc: all real numbers other than $n\pi$ for any integer n

$x \neq 0, \pi, -\pi, 2\pi, -2\pi, \ldots$



Recall that the terminal points (and therefore the trig function values) are the same numerical value with different signs.

Positive Trig Functions



Example 2

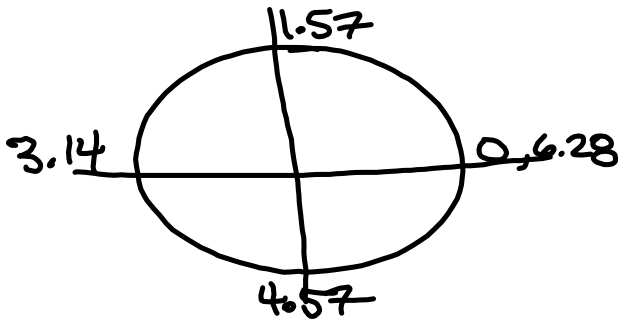
Determine the sign of the following functions.

a) $\cos \frac{\pi}{3}$ positive

↑ QI

b) $\tan 6$ negative

↑ QIV

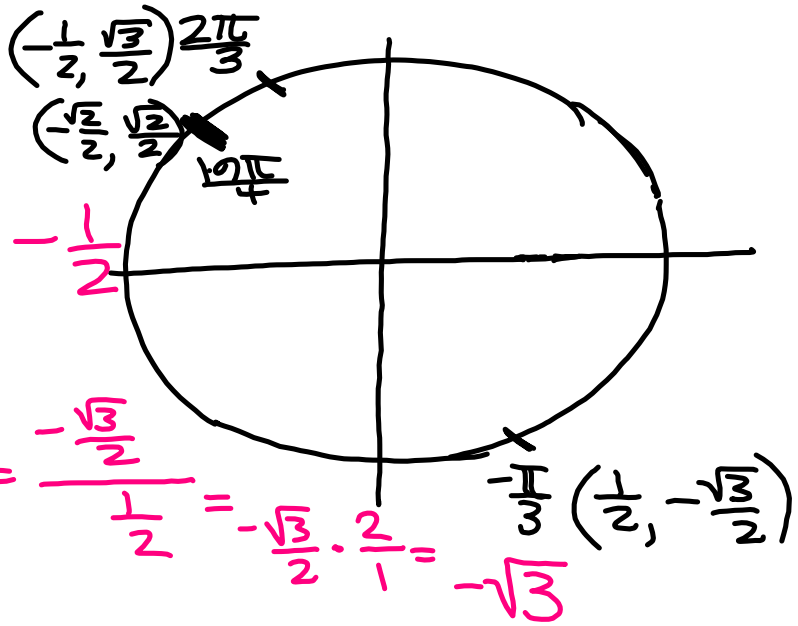


c) If $\cos t < 0$ and $\sin t > 0$, in what quadrant must the terminal point lie? QII

QII, QIII neg. QI QII pos.

Example 3

Find each value.



a) $\cos \frac{2\pi}{3} = -\frac{1}{2}$

b) $\tan\left(-\frac{\pi}{3}\right) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

c) $\sin \frac{19\pi}{4} = \frac{\sqrt{2}}{2}$

$4\frac{3}{4}$

Example 4

The terminal point P(x, y) determined by t is given. Find sin t, cos t, tan t, csc t, sec t, & cot t.

a) $\left(-\frac{3}{5}, \frac{4}{5}\right)$

$\sin t = \frac{4}{5}$ $\quad \csc t = \frac{5}{4}$

$\cos t = -\frac{3}{5}$ $\quad \sec t = -\frac{5}{3}$

$\tan t = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$ $\quad \cot t = -\frac{3}{4}$

b) $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$

$\sin t = -\frac{2\sqrt{2}}{3}$ $\quad \csc t = -\frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

$\cos t = -\frac{1}{3}$ $\quad \sec t = -3$

$\tan t = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = 2\sqrt{2}$ $\quad \cot t = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$