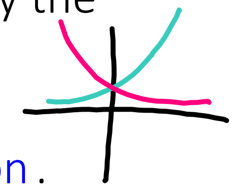
6.3 Logarithmic Functions

passes both VLT & HLT

Every exponential function $y = b^x$ with $b > 0$ and $b \neq 1$ is a one-to-one function by the horizontal line test.

Therefore it has an inverse function, which is called the logarithmic function.



exponential form $b^x = y$

logarithmic form $x = \log_b y$

x equals log base b of y

exponent

base

Example 1

Rewrite each exponential in logarithmic form.

a) $\underline{10}^5 = 100{,}000$

base

$\log_{10} 100{,}000 = 5$

b) $\underline{2}^3 = 8$

base

$\log_2 8 = 3$

c) $\underline{\left(\frac{1}{3}\right)}^2 = \frac{1}{9}$

base

$\log_{\frac{1}{3}} \frac{1}{9} = 2$

Example 2

Rewrite each logarithm in exponential form.

a) $\log_2 32 = 5$ exp

base

$$2^5 = 32$$

b) $\log_5 \left(\frac{1}{25}\right) = -2$ exp

base

$$5^{-2} = \frac{1}{25}$$

c) $\log_7 h = g$ exp

base

$$7^g = h$$

It is important to remember that $\log_a x$ is an exponent!
The answer to every log is the exponent!

One-to-One Property of Exponents

If $b^x = b^y$, then $x = y$.

If base is same on each side, then exponents are equal.

Example 3

Evaluate using the properties of logs.

a) $\log_5 25 = x$ exp. base

$5^x = 25$

$5^x = 5^2$

$\boxed{x=2}$

b) $\log_{10} 1 = x$ exp base

$10^x = 1$

$10^x = 10^0$

$\boxed{x=0}$

c) $\log_3 81 = x$ exp base

$3^x = 81$

$3^x = 3^4$

$\boxed{x=4}$

d) $\log_8 \frac{1}{64} = x$ exp base

$8^x = \frac{1}{64}$

$8^x = \frac{1}{8^2}$

$8^x = 8^{-2}$

$\boxed{x=-2}$

Example 4

Evaluate using the properties of logs.

a) $\log_x 49 = 2$ exp base

$x^2 = 49$

$x^2 = 7^2$

$\boxed{x=7}$

b) $\log_x \frac{1}{16} = -4$ exp base

$x^{-4} = \frac{1}{16}$

$x^{-4} = \frac{1}{2^4}$

$x^{-4} = 2^{-4}$

$\boxed{x=2}$

c) $\log_x \frac{1}{125} = 3$ exp base

$x^3 = \frac{1}{125}$

$x^3 = \frac{1}{5^3} \rightarrow x^3 = \left(\frac{1}{5}\right)^3$

$\boxed{x=\frac{1}{5}}$

d) $\log_x 64 = 1$ exp base

$x^1 = 64^1$

$\boxed{x=64}$

Example 5

Evaluate using the properties of logs.

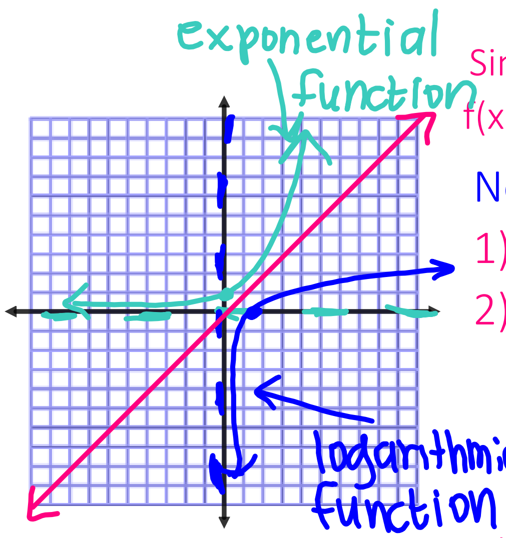
a) $\log_3 x = -3$ exp   base $3^{-3} = x$ $\frac{1}{3^3} = x$ $\boxed{x = \frac{1}{27}}$

b) $\log_5 x = 4$ exp   base $5^4 = x$ $\boxed{x = 625}$

c) $\log_{27} x = \frac{1}{3}$ exp   base $27^{1/3} = x$ $\sqrt[3]{27} = x$ $\boxed{x = 3}$

d) $\log_{10} x = 0$ exp   base $10^0 = x$ $\boxed{x = 1}$

Since the log function is the inverse of the exponential function, it can be graphed by switching the domain and range.



Since $f(x) = a^x$ is a rapidly increasing function, $f(x) = \log_a x$ is a very slowly increasing function.

Notice...

1) that since $a^0 = 1$, then $\log_a 1 = 0$.

2) that since the x-axis is the asymptote for the exponential function, then the y-axis is the asymptote for the log function (unless there is a shift).

3) that the log function is a reflection across the line $y = x$.

$m = 1$ y-int $= 0$

Example 6

Sketch the graph by making a table and plotting points. State the domain, range, and any asymptotes.

$f(x) = \log_2 x$

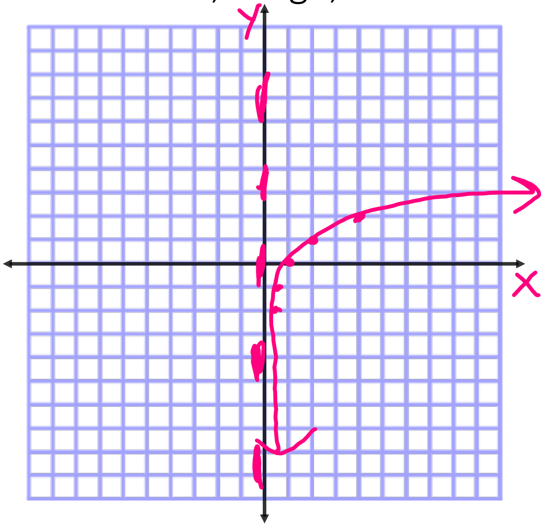
$y = \log_2 x$

exp

base

$2^y = x$

x	y
$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$	-2
$\frac{1}{2} = 2^{-1}$	-1
$1 = 2^0$	0
$2 = 2^1$	1
$4 = 2^2$	2



D: $x > 0$

R: $\mathbb{R}$

Asym: $x = 0$

Example 7

Sketch the graph by plotting points.

State the domain, range, and any asymptotes.

$f(x) = 2 + \log_2 (x - 3)$

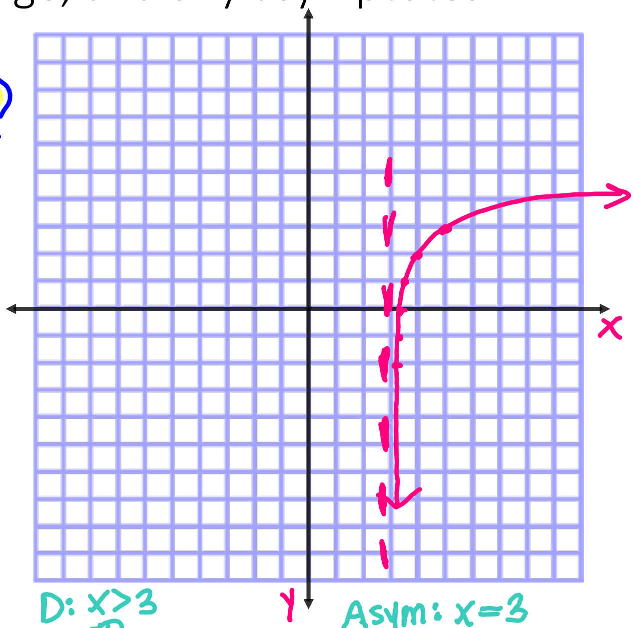
$$y = 2 + \log_2 (x-3)$$
$$\underline{-2 \quad -2}$$
$$y - 2 = \log_2 (x-3)$$

exp

base

$$2^{y-2} = x - 3$$
$$\underline{+3 \qquad +3}$$
$$2^{y-2} + 3 = x$$

x	y
$3\frac{1}{16} = \frac{1}{16} + 3 = 2^{-4} + 3$	-2
$3\frac{1}{8} = \frac{1}{8} + 3 = 2^{-3} + 3$	-1
$3\frac{1}{4} = \frac{1}{4} + 3 = 2^{-2} + 3$	0
$3\frac{1}{2} = \frac{1}{2} + 3 = 2^{-1} + 3$	1
$4 = 1 + 3 = 2^0 + 3$	2
$5 = 2 + 3 = 2^1 + 3$	3



D: $x > 3$

R: $\mathbb{R}$

Asym: $x = 3$

Example 8

Sketch the graph by plotting points.

State the domain, range, and any asymptotes.

$f(x) = -\log_2 x$

look at table for #6

change y-values to opposite

D: $x > 0$

R: $\mathbb{R}$

Asym: $x = 0$