4.5 Write Equations of Parallel & Perpendicular Lines

Remember: In the last chapter we learned that parallel lines have the same slope.

$y = mx + b$

Example 1: Write an equation of the line that passes through (-8,5) and is parallel to the line $y = \frac{3}{4}x - 1$.

$x_1\ y_1$

$m = \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{3}{4}(x + 8)$

$y - 5 = \frac{3}{4}x + 6$

$+5 \qquad +5$

$\boxed{y = \frac{3}{4}x + 11}$

$y = mx + b$

Example 2: Write an equation of the line that passes through (-3,1) and is parallel to the line $3x - 2y = 7$.

$x_1\ y_1$

Solve for y.

$-3x \qquad -3x$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{7}{-2}$

$y = \frac{3}{2}x - \frac{7}{2}$

m

$y - y_1 = m(x - x_1)$

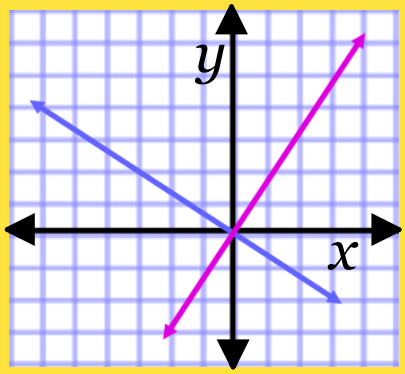
$y - 1 = \frac{3}{2}(x + 3)$

$y - 1 = \frac{3}{2}x + \frac{9}{2}$

$+1 \qquad +1$

$\boxed{y = \frac{3}{2}x + \frac{11}{2}}$

Two lines are perpendicular if they intersect to form a right angle.



$-\frac{2}{3}$ & $\frac{3}{2}$

$\frac{4}{1}$ & $-\frac{1}{4}$

Perpendicular lines have slopes that are opposite reciprocals [change sign] [flip] (or negative reciprocals).

Example 3: Determine which lines, if any, are parallel or perpendicular.

Line *a*: $2x + 6y = -3$ (same slope)

$-2x \quad -2x$

$\frac{6y}{6} = \frac{-2x}{6} - \frac{3}{6}$

$y = -\frac{1}{3}x - \frac{1}{2}$

$m = -\frac{1}{3}$

Line *b*: $y = 3x - 8$ (opp. rec. slopes)

$m = \frac{3}{1}$

a & b are perpendicular

Line *c*: $-1.5x + 4.5y = 6$

$+1.5x \quad +1.5x$

$\frac{4.5y}{4.5} = \frac{1.5x}{4.5} + \frac{6}{4.5}$

$y = \frac{1}{3}x + \frac{4}{3}$

$m_1 = \frac{1}{3}$

Example 4: Write an equation of the line that passes through (4,3) and is perpendicular to the line $y = 4x - 7$.

(x_1, y_1)

$m = 4$ $\quad m_\perp = -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{4}(x - 4)$

$y - 3 = -\frac{1}{4}x + 1$

$+3 \qquad +3$

$\boxed{y = -\frac{1}{4}x + 4}$

Example 5: Write an equation of the line that passes through (5,2) and is perpendicular to the line $y = -\frac{1}{2}x + 4$.

(x_1, y_1)

$m = -\frac{1}{2}$ $\quad m_\perp = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 5)$

$y - 2 = 2x - 10$

$+2 \qquad +2$

$\boxed{y = 2x - 8}$

Example 6: Write an equation of the line that passes through $(\overset{x_1}{4},\overset{y_1}{-1})$ and is perpendicular to the line $7x - 2y = 3$.

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{2}{7}(x - 4)$

$y + 1 = -\frac{2}{7}x + \frac{8}{7}$

$-1 \qquad -1$

$y = -\frac{2}{7}x + \frac{1}{7}$

Solve for y.

$-7x \qquad -7x$

$\frac{-2y}{-2} = \frac{-7x}{-2} + \frac{3}{-2}$

$y = \frac{7}{2}x - \frac{3}{2}$

$m = \frac{7}{2}$ $\quad m_1 = -\frac{2}{7}$

Example 7: Write an equation of the line that passes through $(\overset{x_1}{-2},\overset{y_1}{2})$ and is perpendicular to the line $y = 7$.

$x = -2$

horizontal → want vertical line $x = \#$