6.2 Exponential Functions

An exponential function has the variable as an exponent.

base, exponent

$$f(x) = a \cdot b^x$$

b is any positive number other than 1

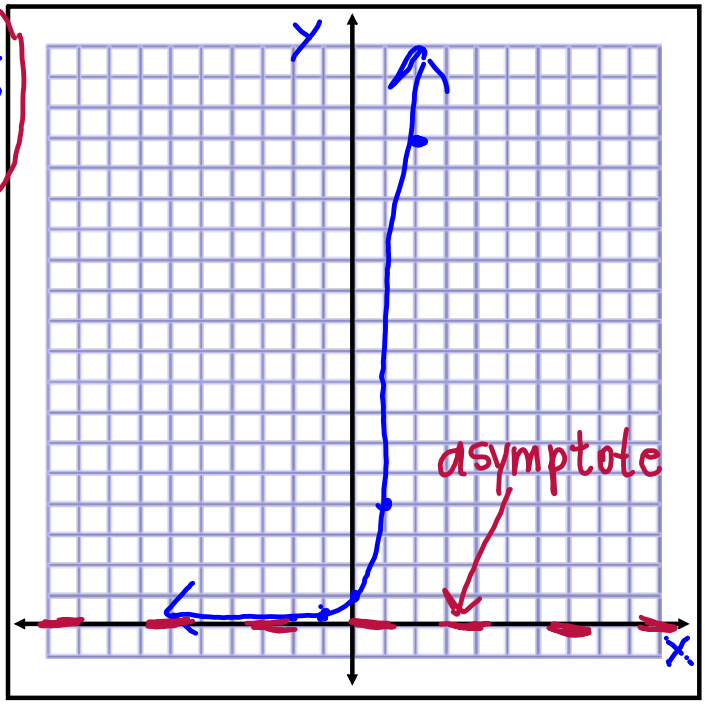
examples: $f(x) = 2^x$ or $f(x) = \left(\frac{1}{2}\right)^x$

Example 1

growth

Make a table of values and graph $y = 4^x$.

x	y
-2	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
-1	$4^{-1} = \frac{1}{4^1} = \frac{1}{4}$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$



Domain: $\mathbb{R}$

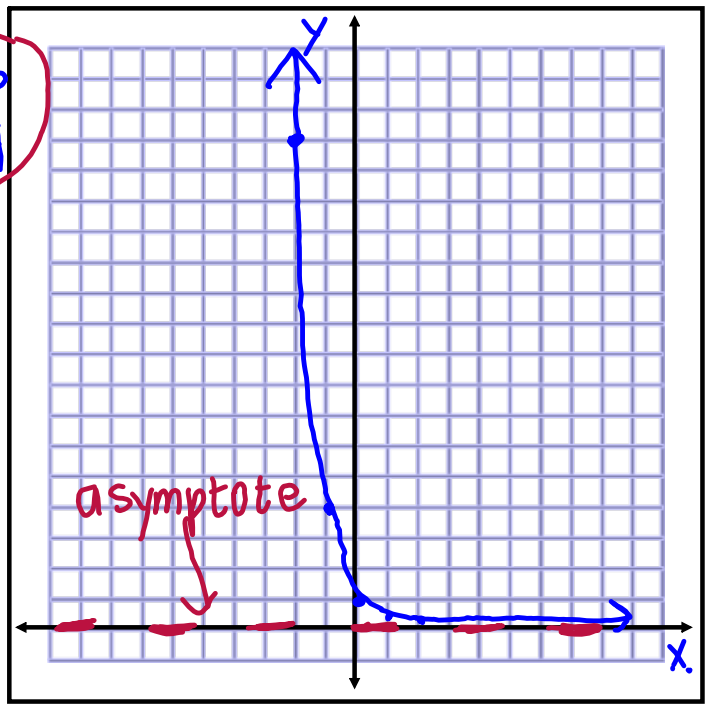
Range: $y > 0$

Example 2

decay

Make a table of values and graph $y = \left(\frac{1}{4}\right)^x$.

x	y
-2	$\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 16$
-1	$\left(\frac{1}{4}\right)^{-1} = \left(\frac{4}{1}\right)^1 = 4$
0	$\left(\frac{1}{4}\right)^0 = 1$
1	$\left(\frac{1}{4}\right)^1 = \frac{1}{4}$
2	$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$



Domain: $\mathbb{R}$

Range: $y > 0$

The exponential function is a growth function if the base is greater than 1.

The exponential function is a decay function if the base is between 0 and 1.

Example 3

Graph each function below. Identify the domain, range, & asympote.

a) $y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

D: ℝ
R: $y > 0$
asym: $y = 0$

b) $y = 2(3)^x$

vert. stretch by a fact. of 2
D: ℝ
R: $y > 0$
asym: $y = 0$

c) $y = -3^x$

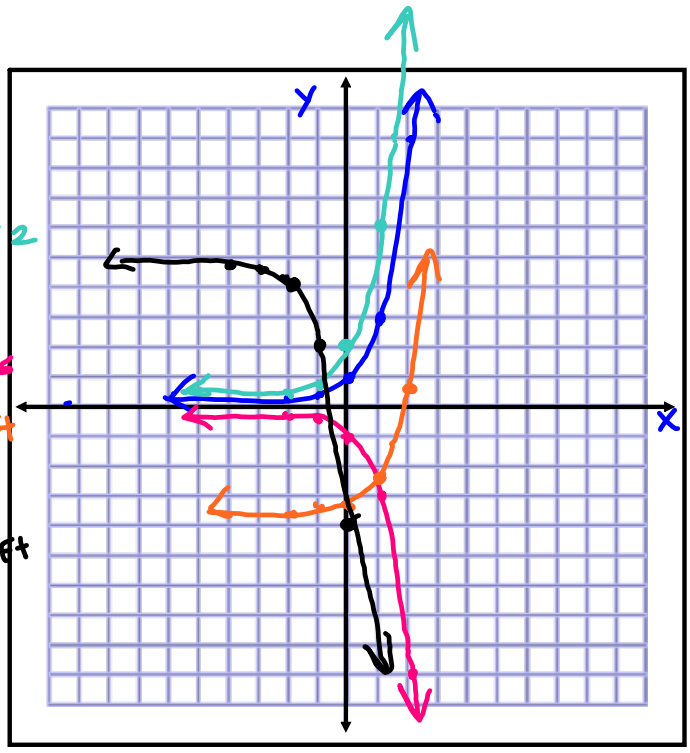
reflection over x-axis
D: ℝ
R: $y < 0$
asym: $y = 0$

d) $y = \frac{1}{2}(3)^x - 4$

vert. shift down 4
comp. by a factor of 1/2
D: ℝ
R: $y > -4$
asym: $y = -4$

e) $y = -3^{x+2} + 5$

left 2
vert. shift up 5
D: ℝ
R: $y < 5$
asym: $y = 5$



Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

A(t) = the amount of $ after t years
P = the amount of $ invested or borrowed
r = percent as a decimal
t = the number of years
n = the number of times interest is compounded per year

n=1 annually
n=2 Semiannually
n=4 quarterly
n=12 monthly

Example 4

A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, and monthly.

$P = 1000$ $t = 3$ $r = 12\% = .12$

annually

$A = 1000\left(1 + \frac{.12}{1}\right)^{1 \cdot 3}$

$A \approx \$1404.93$

quarterly

$A = 1000\left(1 + \frac{.12}{4}\right)^{4 \cdot 3}$

$A \approx \$1425.76$

semiannually

$A = 1000\left(1 + \frac{.12}{2}\right)^{2 \cdot 3}$

$A \approx \$1414.52$

monthly

$A = 1000\left(1 + \frac{.12}{12}\right)^{12 \cdot 3}$

$A \approx \$1430.77$

Example 5

a) Find the final amount for an $800 investment at 7% interest compounded quarterly for 10 years.

b) Find the final amount for a $3000 investment at 4.25% interest compounded daily for 2 years.