6.1 Exponential Growth and Decay

Exponential Growth Equation: $y = a(1 + r)^t$

a is the initial amount

r is the percent increase, written as a decimal

1 + r is the growth factor (or multiplier)

EXAMPLE 1:

$r = 75\% = .75$

In 1995, there were 275 (a) cell phone subscribers in Aiken. The number of subscribers increased by 75% per year after 1995. Write an exponential equation that models the number of cell phone subscribers after *t* years. How many were there in 2004? $t = 9$

$y = 275(1 + .75)^t$

$y = 275(1.75)^9$

$y \approx 42{,}333$ people

EXAMPLE 2:

In the exponential equation

$y = \underset{a}{35}(1.27)^x$, identify:

a) the initial amount,

35

b) the growth factor, $(1+r)$

1.27

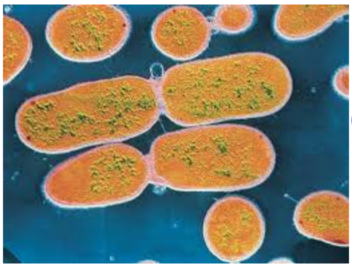
c) the percent increase.

$$1 + r = 1.27$$
$$-1 \qquad -1$$
$$r = .27$$

27%

EXAMPLE 3:

$r = 100\% = 1$



Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If a scientist starts with three bacteria which can double every hour, how many bacteria will she have by the end of the day?

$a = 3$

$$y = 3(1+1)^{24}$$

$$y = 3(2)^{24}$$

doubling

$$y = 50{,}331{,}648 \text{ bacteria}$$

EXAMPLE 4:

In 1970, the population of a city was about 278,000. Since then, the city population has grown at an average annual rate of 1.8%.

a) Write an exponential equation that models the population of this city *t* years after 1970.

$$y = 278{,}000(1+.018)^t$$

b) About how many people lived in the city in 1990? $t = 20$

$$y = 278{,}000(1.018)^{20}$$

397,192 people

c) What is the population of this city today? $2024 \rightarrow t = 54$

$$y = 278{,}000(1.018)^{54}$$

728,492 people

Exponential Decay Equation: $y = a(1 - r)^t$

a is the initial amount

r is the percent decrease, written as a decimal

1 - r is the decay factor (or multiplier)

EXAMPLE 5:

Jolene purchases a new car for $22,499. The value of the car decreases by 11% each year. Write the exponential equation that models the car's value after *t* years. Then find its value after 3 years.

$y = 22{,}499(1 - .11)^t$

$y = 22{,}499(.89)^3$

$15,861.10

EXAMPLE 6:

In the exponential equation $y = 200(0.71)^x$, identify:

a) the initial amount,

200

b) the decay factor, 1-r

.71

c) the percent decrease.

$\cancel{1} - r = .71$

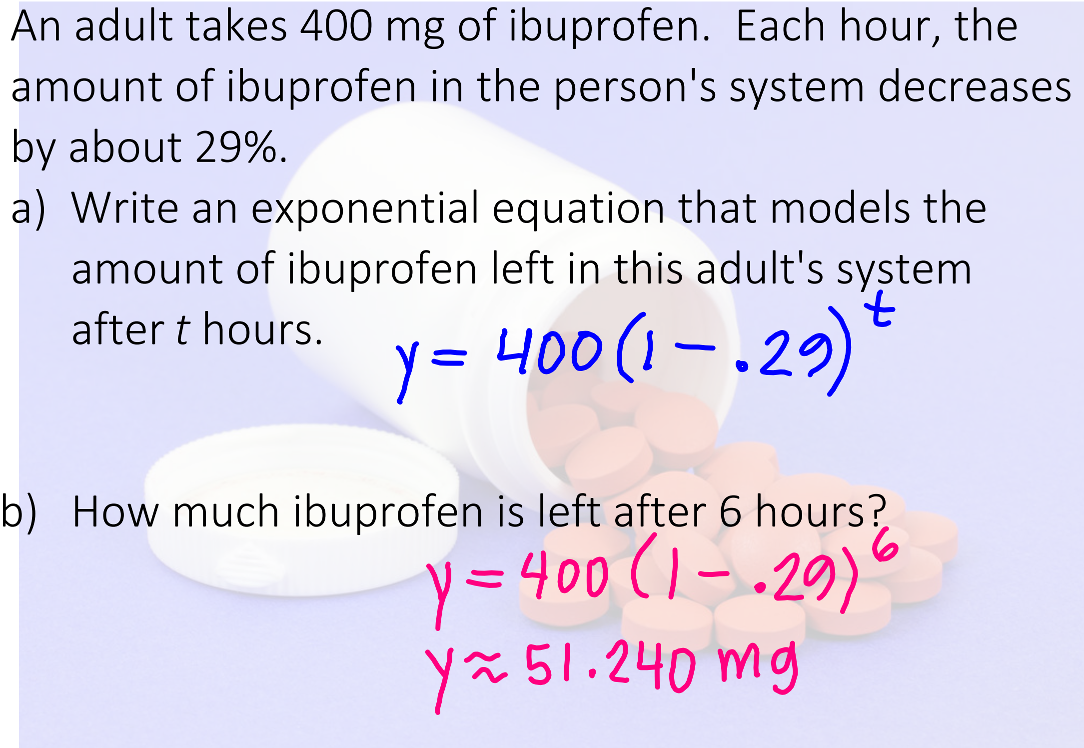
$-\cancel{1} \quad -1$

$-r = -.29$

$\frac{-r}{-1} = \frac{-.29}{-1}$

$r = .29 \rightarrow 29\%$

EXAMPLE 7:



An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%.

a) Write an exponential equation that models the amount of ibuprofen left in this adult's system after *t* hours.

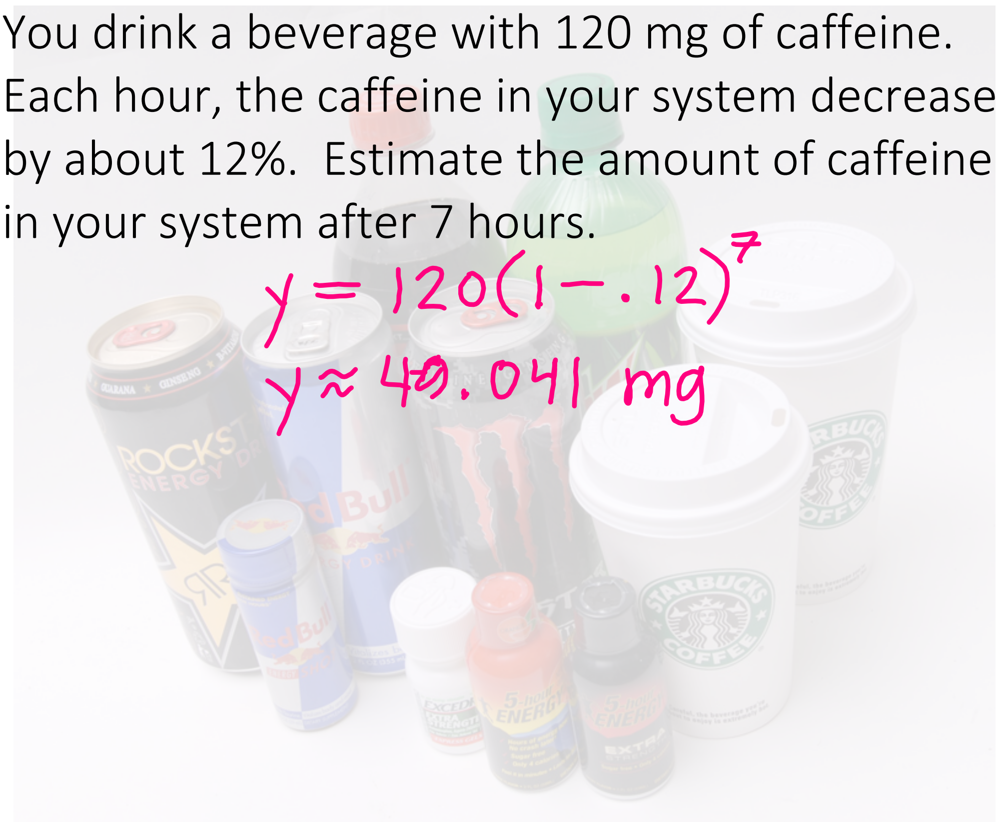
$y = 400(1-.29)^t$

b) How much ibuprofen is left after 6 hours?

$y = 400(1-.29)^6$

$y \approx 51.240$ mg

EXAMPLE 8:



You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. Estimate the amount of caffeine in your system after 7 hours.

$y = 120(1-.12)^7$

$y \approx 49.041$ mg