4.5 Exponential and Logarithmic Equations

Steps for Solving Exponential Equations

1. Isolate the exponential expression on one side.
2. Take the logarithm of each side. Then use laws of logarithms to "bring down" the exponent.
3. Solve for the variable.

Example 1: Solve each equation.

a) $2^x = 7$

$\log 2^{x} = \log 7$

$\dfrac{x \cancel{\log 2}}{\cancel{\log 2}} = \dfrac{\log 7}{\log 2}$

$\boxed{x \approx 2.807}$

b) $3^{x+2} = 7$

$\log 3^{x+2} = \log 7$

$(x+2)\log 3 = \log 7$

$x\log 3 + 2\log 3 = \log 7$

$\quad -2\log 3 \quad -2\log 3$

$\dfrac{x \cancel{\log 3}}{\cancel{\log 3}} = \dfrac{\log 7 - 2\log 3}{\log 3}$

$\boxed{x \approx -0.229}$

c) $8e^{2x} = 20$

$\dfrac{8e^{2x}}{8} = \dfrac{20}{8}$

$e^{2x} = \dfrac{5}{2}$

$\ln e^{2x} = \ln \dfrac{5}{2}$

$\dfrac{\cancel{2}x}{\cancel{2}} = \dfrac{\ln \frac{5}{2}}{2}$

$\boxed{x \approx 0.458}$

Example 2: Solve each equation.

a) $e^{3-2x} = 4$

$\cancel{\ln e}^{3-2x} = \ln 4$

$3 - 2x = \ln 4$
$-3 \qquad\quad -3$

$-2x = \ln 4 - 3$

$\dfrac{-2x}{-2} = \dfrac{\ln 4 - 3}{-2}$

$\boxed{x \approx 0.807}$

b) $2^{3x+1} = 3^{x-2}$

$\log 2^{3x+1} = \log 3^{x-2}$

$(3x+1)\log 2 = (x-2)\log 3$

$3x\log 2 + \log 2 = \cancel{x\log 3} - 2\log 3$
$-x\log 3 \qquad\qquad -\cancel{x\log 3}$

$3x\log 2 + \cancel{\log 2} - x\log 3 = -2\log 3$
$-\cancel{\log 2} \qquad\qquad -\log 2$

$3x\log 2 - x\log 3 = -2\log 3 - \log 2$

$\dfrac{x\cancel{(3\log 2 - \log 3)}}{\cancel{3\log 2 - \log 3}} = \dfrac{-2\log 3 - \log 2}{3\log 2 - \log 3}$

$\boxed{x \approx -2.947}$

Sometimes we have to solve by factoring.

Example 3: Solve each equation.

a) $1 \cdot e^{2x} - 1e^{x} - 6 = 0$

sum -1 product -6

$\dfrac{-3}{1} \quad \dfrac{2}{1}$

$(e^x - 3)(e^x + 2) = 0$

$e^x - 3 = 0$
$+3 \quad +3$

$\cancel{\ln e}^x = \ln 3$

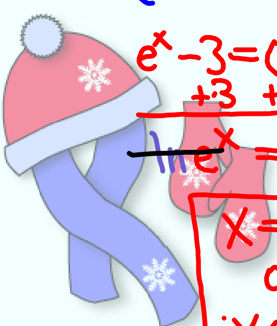
$\boxed{x = \ln 3 \text{ or } x \approx 1.099}$

$e^x + 2 = 0$
$-2 \quad -2$

$\cancel{\ln e}^x = \ln(-2)$

$\cancel{x = \ln(-2)}$

doesn't work



b) $3x^2e^x + x^3e^x = 0$

$x^2e^x(3 + x) = 0$

$x^2 = 0 \quad e^x = 0 \quad 3 + x = 0$

$\boxed{x = 0} \quad \cancel{\ln e}^x = \ln 0 \quad \boxed{x = -3}$

$\cancel{x = \ln 0}$

Steps for Solving Logarithmic Equations

1. Isolate the logarithmic term on one side of the equation. You may need to use the laws of logarithms to first combine the logarithmic terms.
2. Write the equation in exponential form (or raise the base to each side of the equation).
3. Solve for the variable.
4. Check to make sure your solution is not extraneous.

Example 4: Solve each equation.

a) $\log_e x = 8$

$\ln x = 8$

$\rightarrow e^8 = x$

$\boxed{x \approx 2980.958}$

$\cancel{\ln e}^8 = 8$ ✓

b) $\log_2 (25 - x) = 3$

$2^3 = 25 - x$

$8 = \cancel{25} - x$

$-25 \quad \cancel{-25}$

$\frac{-17}{-1} = \frac{-x}{-1}$

$\boxed{17 = x}$

$\log_2(25-17) \stackrel{?}{=} 3$

$\log_2 8 = 3$ ✓

Example 5: Solve each equation.

a) $\cancel{4} + 3 \log (2x) = 16$

$\cancel{-4} \qquad -4$

$\frac{3 \log (2x)}{3} = \frac{12}{3}$

$\log_{10}(2x) = 4$

$10^4 = 2x \longrightarrow \frac{10{,}000}{2} = \frac{2x}{2}$

$\boxed{5{,}000 = x}$

b) $\log (x + 2) + \log (x - 1) = 1$

$\log_{10}(x+2)(x-1) = 1$

$10^1 = (x+2)(x-1)$

$10 = x^2 + x - 2$

$-10 \qquad -10$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x \neq -4 \qquad \boxed{x=3}$

Example 6: Solve each equation graphically.

(Do you remember what else solutions are called?)

$x^2 = 2 \ln(x + 2)$

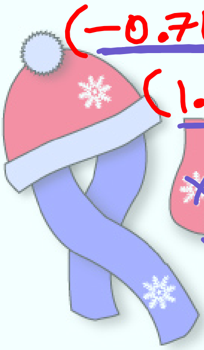
$y = x^2$

$y = 2\ln(x+2)$

Graph & find intersection.

$(\underline{-0.712}, 0.507)$

$(\underline{1.601}, 2.562)$



$x \approx -0.712$

$x \approx 1.601$

$x^2 = 2\ln(x+2)$

$-x^2 \qquad -x^2$

$0 = \underline{2\ln(x+2) - x^2}$

Graph & find zeroes.