4.5 Exponential and Logarithmic Equations

Steps for Solving Exponential Equations

1. Isolate the exponential expression on one side.
2. Take the logarithm of each side. Then use laws of logarithms to "bring down" the exponent.
3. Solve for the variable.

Example 1: Solve each equation.

a) $2^x = 7$

$\log 2^x = \log 7$

$x \log 2 = \log 7$

$\frac{x \log 2}{\log 2} = \frac{\log 7}{\log 2}$

$x \approx 2.807$

b) $3^{x+2} = 7$

$\log 3^{x+2} = \log 7$

$(x+2)\log 3 = \log 7$

$x \log 3 + 2\log 3 = \log 7$

$-2\log 3 \quad -2\log 3$

$\frac{x \log 3}{\log 3} = \frac{\log 7 - 2\log 3}{\log 3}$

$x \approx -0.229$

c) $\frac{8e^{2x}}{8} = \frac{20}{8}$

$e^{2x} = \frac{5}{2}$

$\ln e^{2x} = \ln \frac{5}{2}$

$\frac{2x}{2} = \frac{\ln \frac{5}{2}}{2}$

$x \approx 0.458$

Example 2: Solve each equation.

a) $e^{3-2x} = 4$

$\ln e^{3-2x} = \ln 4$

$3 - 2x = \ln 4$

$-3 \quad -3$

$-2x = \ln 4 - 3$

$-2 \quad -2$

$x \approx 0.807$

b) $2^{3x+1} = 3^{x-2}$

$\log 2^{3x+1} = \log 3^{x-2}$

$(3x+1)\log 2 = (x-2)\log 3$

$3x\log 2 + \log 2 = x\log 3 - 2\log 3$

$-x\log 3 \quad -x\log 3$

$3x\log 2 + \log 2 - x\log 3 = -2\log 3$

$-\log 2 \quad -\log 2$

$3x\log 2 - x\log 3 = -2\log 3 - \log 2$

$x(3\log 2 - \log 3) = -2\log 3 - \log 2$

$3\log 2 - \log 3 \quad 3\log 2 - \log 3$

$x \approx -2.947$

Sometimes we have to solve by factoring.

Example 3: Solve each equation.

a) $1 \cdot e^{2x} - 1e^{x} - 6 = 0$

sum -1 product -6

$-3 \quad 2$

$1 \quad 1$

$(e^x - 3)(e^x + 2) = 0$

$e^x - 3 = 0$ $\quad$ $e^x + 2 = 0$

$+3 \quad +3$ $\quad$ $-2 \quad -2$

$\ln e^x = \ln 3$ $\quad$ $\ln e^x = \ln(-2)$

$x = \ln 3$ $\quad$ $x = \ln(-2)$

or $\quad$ doesn't work

$x \approx 1.099$

b) $3x^2e^x + x^3e^x = 0$

$x^2e^x(3 + x) = 0$

$x^2 = 0 \quad e^x = 0 \quad 3 + x = 0$

$x = 0 \quad \ln e^x = \ln 0 \quad x = -3$

$x = \ln 0$