4.3 Write Linear Equations in Point-Slope Form

You can write an equation of a line if you know its slope and a point from the line.

In this chapter we will learn several forms of equations for lines.

For a given point (x_1, y_1) on a nonvertical line with slope m, the point-slope form of a linear equation is as follows:

$$y - y_1 = m(x - x_1)$$

↑   ↑
stay variables

Example 1

Write the point-slope form of an equation of the line passing through the given point and having the given slope.

a) $(3,5)$, m = $\frac{4}{3}$
   $x_1 y_1$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{4}{3}(x - 3)$$

b) $(-2,0)$, m = $-\frac{3}{2}$
   $x_1 y_1$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{3}{2}(x + 2)$$

c) $(-3,2)$, m = -4
   $x_1 y_1$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -4(x + 3)$$

Example 2

Write the point-slope form of an equation of the line passing through the given points.

a) $(-3,6)$, $(-5,9)$

$$x_1 \ y_1 \qquad x_2 \ y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-6}{-5+3} = \frac{3}{-2}$$

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -\frac{3}{2}(x + 3)$$

b) $(14,3)$, $(-11,3)$

$$x_1 \ y_1 \qquad x_2 \ y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{-11-14} = \frac{0}{-25} = 0$$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 0(x - 14)$$
$$y - 3 = 0$$

Example 3

Give the slope of each line and name a point on the line.

$$y - y_1 = m(x - x_1) \qquad (x_1, y_1)$$

a) $y - 4 = -3(x + 1)$

$$y - 4 = -3(x - -1)$$

$$m = -3$$
$$(-1, 4)$$

b) $y + 6 = \frac{1}{4}(x - 8)$

$$y - -6 = \frac{1}{4}(x - 8)$$

$$m = \frac{1}{4}$$
$$(8, -6)$$

c) $y = -\frac{1}{2}(x + 9)$

$$y - 0 = -\frac{1}{2}(x - -9)$$

$$m = -\frac{1}{2}$$
$$(-9, 0)$$

Example 4

Graph the equations below.

a) $y + 2 = 3(x - 4)$ m=$\frac{3}{1}$ b) $y - 3 = -\frac{2}{3}(x -)$ m=$-\frac{2}{3}$

$y - -2 = 3(x - 4)$ (4,-2)   (0, 3)
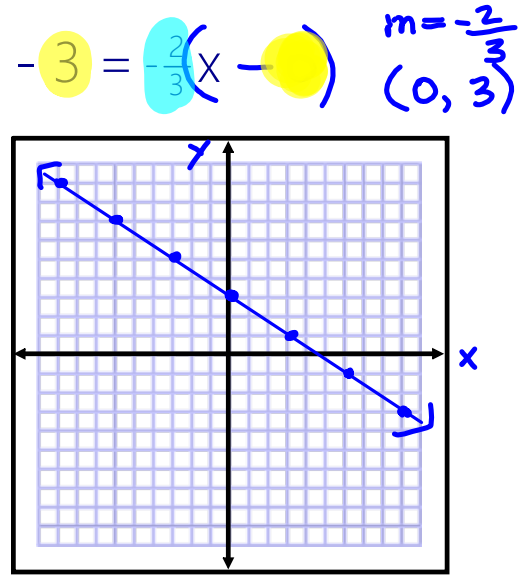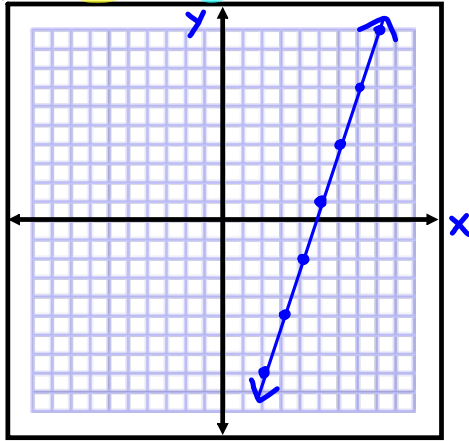
Example 4 (continued)

Graph the equations below.

c) $y = -4(x + 5)$ m=-4   d) $y + 6 = \frac{4}{5}(x + 7)$ m=$\frac{4}{5}$

$y - 0 = -4(x - -5)$ (-5,0) $y - -6 = \frac{4}{5}(x - -7)$ (-7,-6)
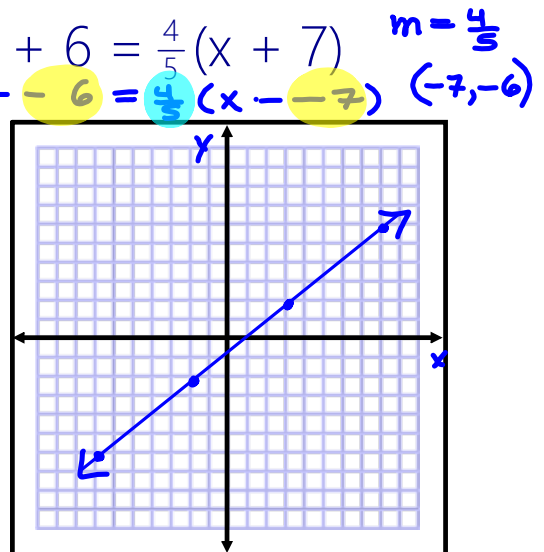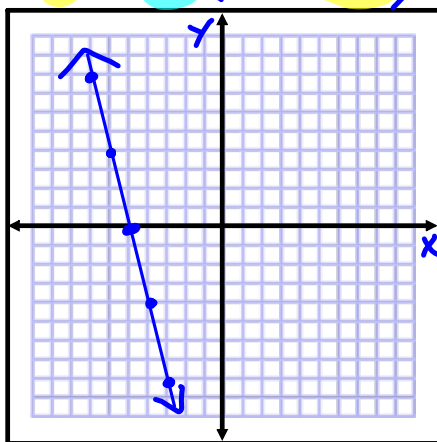
Example 5

Write an equation in point-slope form of the lines graphed below (use the right hand point).

a)

$x_1 \; y_1$
$(4,1)$
$+7$
$+4$
$m = \frac{4}{7}$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{4}{7}(x - 4)$$

b)
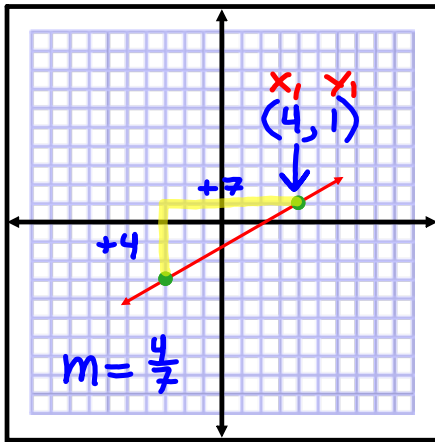
$x_1 \; y_1$
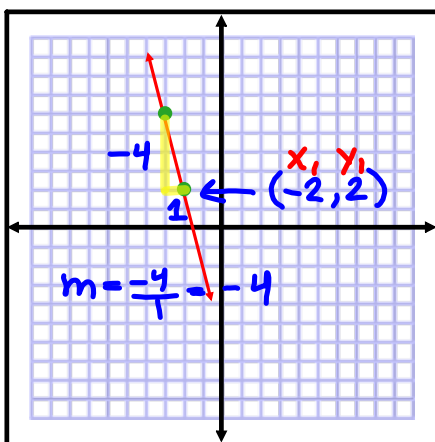$(2,2)$
-2
$+5$
$m = -\frac{2}{5}$
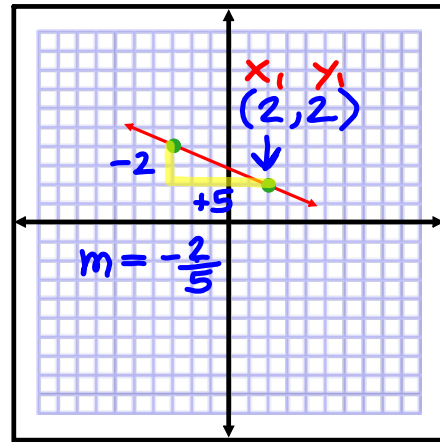
$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{2}{5}(x - 2)$$

Example 5 (continued)

Write an equation in point-slope form of the lines graphed below (use the right hand point).
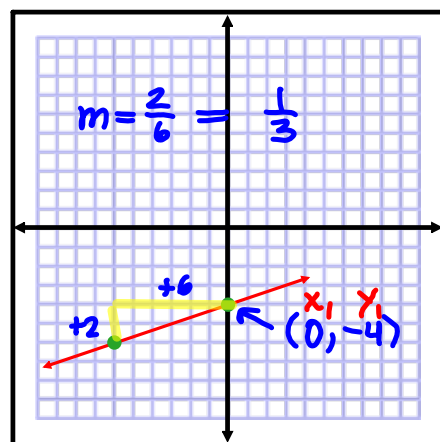
c)

-4
$x_1 \; y_1$
$\leftarrow (-2, 2)$
1
$m = \frac{-4}{1} = -4$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -4(x + 2)$$

d)

$m = \frac{2}{6} = \frac{1}{3}$

$+6$
$+2$
$x_1 \; y_1$
$\leftarrow (0, -4)$

$$y - y_1 = m(x - x_1)$$
$$y + 4 = \frac{1}{3}(x - 0)$$