

5.5 Part 1 Completing the Square

Reminder: Factor each below.

1. $x^2 + 14x + 49$

| Sum 14 | product 49 |
|--------------|----------------|
| $1 \cdot 49$ | $-1 \cdot -49$ |
| $7 \cdot 7$ | $-7 \cdot -7$ |

$$\frac{7}{1} \quad \frac{7}{1}$$
$$(x+7)(x+7)$$

or

$$(x+7)^2$$

2. $x^2 - 8x + 16$

| Sum -8 | product 16 |
|--------------|----------------|
| $1 \cdot 16$ | $-1 \cdot -16$ |
| $2 \cdot 8$ | $-2 \cdot -8$ |
| $4 \cdot 4$ | $-4 \cdot -4$ |

$$\frac{-4}{1} \quad \frac{-4}{1}$$
$$(x-4)(x-4)$$

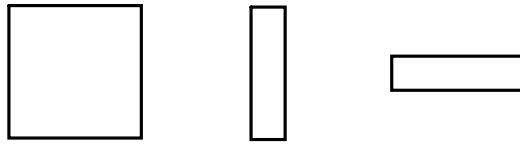
or

$$(x-4)^2$$

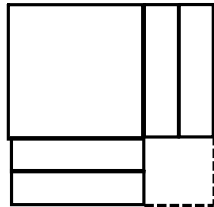
Find the value of c so you can factor as a **perfect square trinomial**.

3. $x^2 - 20x + c$

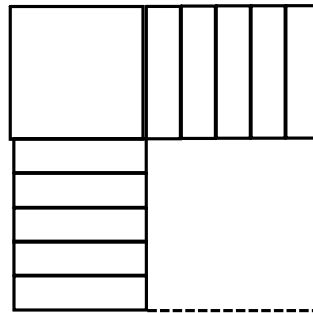
4. $x^2 + 16x + c$



$$x^2 + 4x + \underline{\hspace{2cm}}$$



$$x^2 + 10x + \underline{\hspace{2cm}}$$



COMPLETING THE SQUARE

- STEP 1: Move the constant to the other side. ≠ (no variable)
- STEP 2: Find $\frac{1}{2}$ of b . $ax^2 + bx + c$
- STEP 3: Square the result of Step 2.
- STEP 4: Add the result of Step 3.
- STEP 5: Factor and solve by square roots.

Important: The coefficient of x^2 must be 1!

Solve by completing the square.

$$\begin{array}{l}
 5. \quad x^2 + 6x - 16 = 0 \\
 \quad \quad \quad +16 \quad +16 \\
 \hline
 x^2 + 6x + 9 = 16 + 9 \\
 \frac{1}{2}(6) = 3 \\
 (3)^2 = 9 \\
 \sqrt{(x+3)^2} = \sqrt{25} \\
 x+3 = \pm 5 \\
 \quad -3 \quad -3 \\
 \hline
 x = -3 \pm 5 \\
 \swarrow \quad \searrow \\
 x = -3 + 5 \quad x = -3 - 5 \\
 \boxed{x = 2} \quad \boxed{x = -8}
 \end{array}$$

$$\begin{array}{l}
 6. \quad x^2 + 2x = -8x \\
 \quad \quad \quad +8x - 25 \quad +8x - 25 \\
 \hline
 x^2 + 8x + 16 = -25 + 16 \\
 \frac{1}{2}(8) = 4 \\
 (4)^2 = 16 \\
 \sqrt{(x+4)^2} = \sqrt{-9} \\
 x+4 = \pm 3i \\
 \quad -4 \quad -4 \\
 \hline
 \boxed{x = -4 \pm 3i}
 \end{array}$$

Solve by completing the square.

$$\begin{array}{l}
 7. \quad x^2 = -12x - 4 \\
 \quad \quad \quad +12x \quad +12x \\
 \hline
 x^2 + 12x + 36 = -4 + 36 \\
 \frac{1}{2}(12) = 6 \\
 (6)^2 = 36 \\
 \sqrt{(x+6)^2} = \sqrt{32} \\
 x+6 = \pm 4\sqrt{2} \\
 \quad -6 \quad -6 \\
 \hline
 \boxed{x = -6 \pm 4\sqrt{2}}
 \end{array}$$

$$\begin{array}{l}
 8. \quad x^2 - 6x + 12 = 0 \\
 \quad \quad \quad -12 \quad -12 \\
 \hline
 x^2 - 6x + 9 = -12 + 9 \\
 \frac{1}{2}(-6) = -3 \\
 (-3)^2 = 9 \\
 \sqrt{(x-3)^2} = \sqrt{-3} \\
 x-3 = \pm i\sqrt{3} \\
 \quad +3 \quad +3 \\
 \hline
 \boxed{x = 3 \pm i\sqrt{3}}
 \end{array}$$

Solve by completing the square.

9. $x^2 = 1 - 10x$

10. $x^2 - 18x = -13$

Solve by completing the square.

11. $x^2 = -40x - 280$

12. $x^2 + 8x + 28 = 0$

Handwritten solution for problem 12:

$$\begin{array}{r} \begin{array}{r} 2 \overline{) 12} \\ \underline{4} \\ 8 \\ \underline{6} \\ 2 \end{array} \\ \hline x^2 + 8x + 28 = -28 + 28 \\ \hline x^2 + 8x + 16 = -28 + 16 \\ \frac{1}{2}(8) = 4 \\ (4)^2 = 16 \\ \sqrt{(x+4)^2} = \sqrt{-12} \\ x + 4 = \pm 2i\sqrt{3} \\ \underline{-4} \quad \underline{-4} \\ \hline \boxed{x = -4 \pm 2i\sqrt{3}} \end{array}$$