4.4 Laws of Logarithms

Product Law $\log_b m + \log_b n = \log_b mn$

Quotient Law $\log_b m - \log_b n = \log_b \frac{m}{n}$

Power Law $n \log_b m = \log_b m^n$

I. Evaluate the Expression

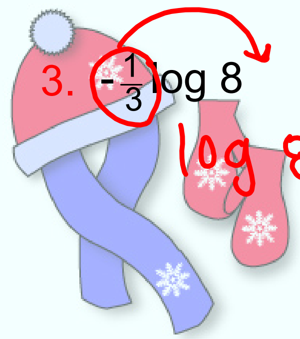
Examples:

1. $\log_4 2 + \log_4 32$

$\log_4 (2 \cdot 32) = \log_4 64 = \cancel{\log_4} 4^3 = 3$

2. $\log_2 80 - \log_2 5$

$\log_2 \left(\frac{80}{5}\right) = \log_2 16 = \cancel{\log_2} 2^4 = 4$



3. $-\frac{1}{3} \log 8$

$\log 8^{-1/3} = \log \frac{1}{8^{1/3}} = \log \frac{1}{\sqrt[3]{8}} = \log \frac{1}{2}$

II. Write in Expanded Form
(sum and/or difference of 2 or more logarithms)

Examples:

1. $\log_6 \dfrac{5x^3}{y}$

$\log_6 5 + \log_6 x^3 - \log_6 y$

$\log_6 5 + 3\log_6 x - \log_6 y$

2. $\log 3x^4$

$\log 3 + \log x^4$

$\log 3 + 4\log x$

3. $\log_7 \dfrac{3x^2}{5y^3}$

$\log_7 3 + \log_7 x^2 - \log_7 5 - \log_7 y^3$

$\log_7 3 + 2\log_7 x - \log_7 5 - 3\log_7 y$

4. $\log_3 \sqrt{5}$

$\log_3 5^{1/2}$

$\frac{1}{2}\log_3 5$

5. $\ln\left(\dfrac{ab}{\sqrt{c}}\right)$

$\ln a + \ln b - \ln c^{1/2}$

$\ln a + \ln b - \frac{1}{2}\ln c$

6. $\log_8 (x^3y^6)$

$\log_8 x^3 + \log_8 y^6$

$3\log_8 x + 6\log_8 y$

III. Write in Condensed Form (single logarithm)

Examples:

1. $\log_8 12 - \log_8 4$

$\log_8 \frac{12}{4} = \log_8 3$

2. $\log_3 4 + \log_3 12 - \log_3 8$

$\log_3 \frac{4 \cdot 12}{8} = \log_3 6$

3. $\log_7 3x - \log_7 9x + \log_7 6y$

$\log_7 \frac{3x \cdot 6y}{9x} = \log_7 \frac{18xy}{9x} = \log_7 2y$

4. $\log_4 10 - 2\log_4 5$

$\log_4 10 - \log_4 5^2 = \log_4 10 - \log_4 25 = \log_4 \frac{10}{25} = \log_4 \frac{2}{5}$

5. $6\log a + 2\log b - 3\log c$

$\log a^6 + \log b^2 - \log c^3 = \log \frac{a^6b^2}{c^3}$

PRACTICE

Evaluate.

1. $\log 25 + \log 40$

$\log 25\cdot 40$

$\log 1000$

$\cancel{\log_{10} 10^3} = \boxed{3}$

Condense.

2. $\log_5 15 - \log_5 1/3$

$\log_5 \frac{15}{1/3}$

$\boxed{\log_5 45}$

3. $\log_8 20 + \log_8 3 - \log_8 4$

$\log_8 \frac{20\cdot 3}{4}$

$\boxed{\log_8 15}$

4. $7 \log_3 y - 4 \log_3 x$

$\log_3 y^7 - \log_3 x^4$

$\boxed{\log_3 \frac{y^7}{x^4}}$

5. $3 \log_2 x + \log_2 6 - 2 \log_2 x$

$\log_2 x^3 + \log_2 6 - \log_2 x^2$

$\log_2 \frac{6x^3}{x^2} = \boxed{\log_2 6x}$

Expand.

6. $\log_9 \frac{2x^3}{3}$

$\log_9 2 + \log_9 x^3 - \log_9 3$

$\log_9 2 + 3\log_9 x - \cancel{\log_9 9^{1/2}}$

$\boxed{\log_9 2 + 3\log_9 x - \frac{1}{2}}$

7. $\log \sqrt{a\sqrt[3]{b}}$

$\log a^{1/2}(b^{1/3})^{1/2}$

$\log a^{1/2} b^{1/6}$

$\log a^{1/2} + \log b^{1/6}$

$\boxed{\frac{1}{2}\log a + \frac{1}{6}\log b}$

WARNING! WARNING! WARNING! WARNING!

Although the laws of logarithms tell us how to compute the logarithm of a product or a quotient, *there is no corresponding rule for the logarithm of a sum or a difference*.

$$\log_a (x + y) \neq \log_a x + \log_a y$$

Also, don't improperly simplify quotients or powers of logarithms. For instance,

$$\frac{\log 6}{\log 2} \neq \log\left(\frac{6}{2}\right) \neq \log 6 - \log 2$$

$$(\log_2 x)^3 \neq 3 \log_2 x$$

IV. Change-of-Base Formula

If a, b, & c are positive numbers with b ≠ 1 and c ≠ 1 then:

num.

$$\log_c a = \frac{\log_b a}{\log_b c}$$

den.

we pick

This allows us to change to a base of 10 so that we can use a calculator!

IV. Change-of-Base Formula

Examples:

num.

1. Evaluate $\log_4 7$. Round to the nearest thousandth.

den.

$$\frac{\log_{10} 7}{\log_{10} 4} \approx 1.404$$

2. Evaluate $\log_6 3$. Round to the nearest thousandth.

$$\frac{\log 3}{\log 6} \approx 0.613$$

3. Evaluate $\log_9 2$. Round to the nearest thousandth.

$$\frac{\log 2}{\log 9} \approx 0.315$$

4. Evaluate $\log_5 8$. Round to the nearest thousandth.

$$\frac{\log 8}{\log 5} \approx 1.292$$