4.5 Write Equations of Parallel & Perpendicular Lines

Remember: In the last chapter we learned that parallel lines have the same slope.

Example 1: Write an equation of the line that passes through (-8,5) and is parallel to the line $y = \frac{3}{4}x - 1$.

$m = \frac{3}{4}$

(x_1, y_1)

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{3}{4}(x + 8)$

$y - 5 = \frac{3}{4}x + 6$

$+5 \quad +5$

$y = \frac{3}{4}x + 11$

$\frac{3}{4} \cdot \frac{2}{1}$

Example 2: Write an equation of the line that passes through (-3,1) and is parallel to the line $3x - 2y = 7$.

(x_1, y_1)

Solve for y: $-3x \quad -3x$

$-2y = -3x + 7$

$-2 \quad -2 \quad -2$

$y = \frac{3}{2}x - \frac{7}{2}$

$m = \frac{3}{2}$

$\frac{3}{2} \cdot \frac{3}{1}$

$y - y_1 = m(x - x_1)$

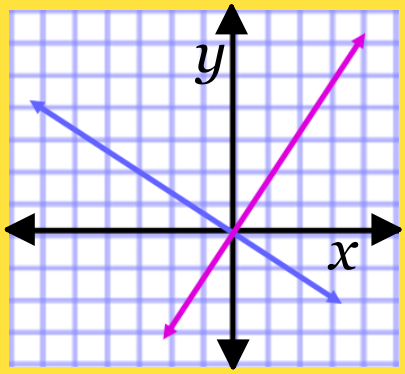
$y - 1 = \frac{3}{2}(x + 3)$

$y - 1 = \frac{3}{2}x + \frac{9}{2}$

$+1 \quad +1 + \frac{2}{2}$

$y = \frac{3}{2}x + \frac{11}{2}$

Two lines are perpendicular if they intersect to form a right angle.



Perpendicular lines have slopes that are opposite reciprocals (or negative reciprocals).

-4 & $\frac{1}{4}$ $\frac{2}{3}$ & $-\frac{3}{2}$

Example 3: Determine which lines, if any, are parallel or perpendicular.

Line *a*: $2x + 6y = -3$ Line *b*: $y = 3x - 8$ Line *c*: $-1.5x + 4.5y = 6$

$-2x \quad -2x$

$\frac{6y}{6} = \frac{-2x}{6} - \frac{3}{6}$

$y = -\frac{1}{3}x - \frac{1}{2}$

$m = -\frac{1}{3}$

$m = 3$

line a & line b are perpendicular

$+1.5x \quad +1.5x$

$\frac{4.5y}{4.5} = \frac{1.5x}{4.5} + \frac{6.0}{4.5}$

$y = \frac{15}{45}x + \frac{60}{45}$

$y = \frac{1}{3}x + \frac{4}{3}$

$m = \frac{1}{3}$

Example 4: Write an equation of the line that passes through $(4,3)$ (x_1, y_1) and is perpendicular to the line $y = 4x - 7$.

$m = 4$

$m_\perp = -\frac{1}{4}$ ← perpendicular symbol

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{4}(x - 4)$

$y - 3 = -\frac{1}{4}x + 1$

$+3 \qquad +3$

$y = -\frac{1}{4}x + 4$

Example 5: Write an equation of the line that passes through $(5,2)$ (x_1, y_1) and is perpendicular to the line $y = -\frac{1}{2}x + 4$.

$m = -\frac{1}{2}$

$m_\perp = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 5)$

$y - 2 = 2x - 10$

$+2 \qquad +2$

$y = 2x - 8$

Example 6: Write an equation of the line that passes through $(4,-1)$ and is perpendicular to the line $7x - 2y = 3$.

(x_1, y_1)

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{2}{7}(x - 4)$

$y + 1 = -\frac{2}{7}x + \frac{8}{7}$

$-1 \qquad -1 \cdot \frac{7}{7}$

$y = -\frac{2}{7}x + \frac{1}{7}$

$-7x \qquad -7x$

$\frac{-2y}{-2} = \frac{-7x}{-2} + \frac{3}{-2}$

$y = \frac{7}{2}x - \frac{3}{2}$

$m = \frac{7}{2}$

$m_\perp = -\frac{2}{7}$

Example 7: Write an equation of the line that passes through $(-2,2)$ and is perpendicular to the line $y = 7$.

(x_1, y_1)

$y = 7$ ↓ horizontal line

$\perp \rightarrow$ vertical line

$x = -2$