7.5 Part 2 Writing Polynomial Functions

Complex Conjugate Root Theorem

If P is a polynomial function with real-number coefficients and a + bi (where b ≠ 0) is a root of P(x) = 0, then a - bi is also a root of P(x) = 0.

In other words...

Complex zeros always come in conjugate pairs.

$5-2i$ & $5+2i$ $\qquad$ $4i$ & $-4i$

Example 1

Write a polynomial function, P, in factored form and in standard form by using the given information.

P is of degree 2; zeros: 4, -2

quadratic

Factored $P(x) = (x-4)(x+2)$

$P(x) = x^2 + 2x - 4x - 8$

standard $P(x) = x^2 - 2x - 8$

Example 2

Write a polynomial function, P, in factored form and in standard form by using the given information.

P is of degree 2; P(0) = 15; zeros: -3, 1

quadratic

$x=0$ $y=15$ unknown coefficient

$P(x) = a(x+3)(x-1)$

$15 = a(0+3)(0-1)$

$15 = a(3)(-1)$

$\frac{15}{-3} = \frac{-3a}{-3}$

$-5 = a$

factored $P(x) = -5(x+3)(x-1)$

$P(x) = -5(x^2 - x + 3x - 3)$

$P(x) = -5(x^2 + 2x - 3)$

standard $P(x) = -5x^2 - 10x + 15$

Example 3

Write a polynomial function, P, in factored form and in standard form by using the given information.

P is of degree 3 (cubic); P(0) = -4; zeros: -1 (mult. of 2), 2

$x=0$ $y=-4$

$P(x) = a(x+1)(x+1)(x-2)$

$-4 = a(0+1)(0+1)(0-2)$

$-4 = a(1)(1)(-2)$

$\frac{-4}{-2} = \frac{-2a}{-2}$

$2 = a$

factored $P(x) = 2(x+1)(x+1)(x-2)$

$P(x) = (2x+2)(x^2 - x - 2)$

$P(x) = 2x(x^2 - x - 2) + 2(x^2 - x - 2)$

$P(x) = 2x^3 - 2x^2 - 4x + 2x^2 - 2x - 4$

standard $P(x) = 2x^3 - 6x - 4$

Example 4

Write a polynomial function, P, in factored form and in standard form by using the given information.

P is of degree 4 (quartic); P(0) = 6 (x=0, y=6); zeros: 3, -4, i, $-i$

$P(x) = a(x-3)(x+4)(x-i)(x+i)$

$6 = a(0-3)(0+4)(0-i)(0+i)$

$6 = a(-3)(4)(-i)(i)$

$6 = 12(-1)a$

$6 = -12a$

$\frac{6}{-12} = \frac{-12a}{-12}$

$-\frac{1}{2} = a$

factored $P(x) = -\frac{1}{2}(x-3)(x+4)(x-i)(x+i)$

$P(x) = -\frac{1}{2}(x^2+x-12)(x^2-i^2)$

$P(x) = -\frac{1}{2}(x^2+x-12)(x^2+1)$

$P(x) = -\frac{1}{2}\left[x^2(x^2+x-12)+1(x^2+x-12)\right]$

$P(x) = -\frac{1}{2}\left[x^4+x^3-12x^2+x^2+x-12\right]$

standard $P(x) = -\frac{1}{2}x^4 - \frac{1}{2}x^3 + \frac{11}{2}x^2 - \frac{1}{2}x + 6$

Example 5

Write a polynomial function, P, in factored form and in standard form by using the given information.

P is of degree 3 (cubic); P(0) = 2 (x=0, y=2); zeros: -5, $2i$, $-2i$

$i^2 = -1$

$P(x) = a(x+5)(x-2i)(x+2i)$

$2 = a(0+5)(0-2i)(0+2i)$

$2 = a(5)(-2i)(2i)$

$2 = -20i^2 a$

$2 = -20(-1)a$

$\frac{2}{20} = \frac{20a}{20}$

$\frac{1}{10} = a$

factored $P(x) = \frac{1}{10}(x+5)(x-2i)(x+2i)$

$(x^2 + 2ix - 2ix - 4i^2)$ (-1)

$P(x) = \frac{1}{10}(x+5)(x^2+4)$

$P(x) = \frac{1}{10}(x^3+4x+5x^2+20)$

standard $P(x) = \frac{1}{10}x^3 + \frac{1}{2}x^2 + \frac{2}{5}x + 2$