5.4 Operations with Complex Numbers

Square Root of Negative Numbers

The square root of a negative real number has TWO imaginary roots: one positive, one negative.

$$\sqrt{-r} = \sqrt{-1} \cdot \sqrt{r} \quad \left(\text{where } \sqrt{-1} = i \right) \quad = i\sqrt{r}$$

and

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$\boxed{i^2 = -1}$

Examples: Simplify.

1. $\sqrt{-81} = 9i$

2. $\sqrt{-48}$

$2 \,|\, 48$, $2 \,|\, 24$, $2 \,|\, 12$, $2 \,|\, 6$, 3

$\sqrt{-1 \cdot \boxed{2 \cdot 2} \cdot \boxed{2 \cdot 2} \cdot 3}$

$4i\sqrt{3}$

3. $\sqrt{-120}$

$2 \,|\, 120$, $2 \,|\, 60$, $2 \,|\, 30$, $3 \,|\, 15$, 5

$\sqrt{-1 \cdot \boxed{2 \cdot 2} \cdot 2 \cdot 3 \cdot 5}$

$2i\sqrt{30}$

4. $\sqrt{-256}$

$16i$

Examples: Solve by taking square roots.

5. $x^2 + 16 = 0$

$-16 \quad -16$

$\sqrt{x^2} = \sqrt{-16}$

$x = \pm 4i$

7. $-3x^2 - 10 = 44$

$+10 \quad +10$

$\frac{-3x^2}{-3} = \frac{54}{-3}$

$\sqrt{x^2} = \sqrt{-18}$

$x = \pm 3i\sqrt{2}$

$2 \cdot \boxed{3 \cdot 3}$

6. $2x^2 + 68 = 20$

$-68 \quad -68$

$\frac{2x^2}{2} = \frac{-48}{2}$

$\sqrt{x^2} = \sqrt{-24}$

$x = \pm 2i\sqrt{6}$

8. $\frac{1}{4}x^2 + 10 = -15$

$-10 \quad -10$

$\frac{\frac{1}{4}x^2}{\frac{1}{4}} = \frac{-25}{\frac{1}{4}}$

$\sqrt{x^2} = \sqrt{-100}$

$x = \pm 10i$

Examples: Solve by taking square roots.

9. $2(x - 1)^2 + 12 = 0$

$-12 \quad -12$

$\frac{2(x-1)^2}{2} = \frac{-12}{2}$

$\sqrt{(x-1)^2} = \sqrt{-6}$

$x - 1 = \pm i\sqrt{6}$

$+1 \quad +1$

$x = 1 \pm i\sqrt{6}$

11. $-5(x + 2)^2 - 7 = 38$

$+7 \quad +7$

$\frac{-5(x+2)^2}{-5} = \frac{45}{-5}$

$\sqrt{(x+2)^2} = \sqrt{-9}$

$x + 2 = \pm 3i$

$-2 \quad -2$

$x = -2 \pm 3i$

10. $\frac{1}{2}(x + 4)^2 - 8 = -26$

$+8 \quad +8$

$\frac{2}{1} \cdot \frac{1}{2}(x+4)^2 = -18 \cdot \frac{2}{1}$

$\sqrt{(x+4)^2} = \sqrt{-36}$

$x + 4 = \pm 6i$

$-4 \quad -4$

$x = -4 \pm 6i$

12. $-\frac{1}{3}(x - 7)^2 + 5 = 23$

$-5 \quad -5$

$\frac{-\frac{1}{3}(x-7)^2}{-\frac{1}{3}} = \frac{18}{-\frac{1}{3}}$

$\sqrt{(x-7)^2} = \sqrt{-54}$

$x - 7 = \pm\sqrt{-2 \cdot 3 \cdot \boxed{3 \cdot 3}}$

$x - 7 = \pm 3i\sqrt{6}$

$+7 \quad +7$

$x = 7 \pm 3i\sqrt{6}$

The standard form of a complex number is

$$a + bi$$

real part — imaginary part

Every number can be written as a <u>complex number</u>.

$9 + 2i \longrightarrow$ imaginary number

$0 + 2i \longrightarrow$ pure imaginary number

$9 + 0i \longrightarrow$ real number

Adding and Subtracting Complex Numbers

Add or subtract:	real part to real part
	imaginary part to imaginary part

<u>Examples</u>: Simplify.

13. $(4 - 1i) + (3 + 2i)$

$7 + 1i$

14. $(7 - 5i) - (1 - 5i)$

$7 \cancel{-5i} - 1 \cancel{+5i}$

6

15. $6 - (-2 + 9i) + (-8 + 4i)$

$\cancel{6} \cancel{+2} - 9i + \cancel{-8} + 4i$

$-5i$

16. $2i - (3 + i) + (2 - 3i)$

$2i - 3 - i + 2 - 3i$

$-1 - 2i$

Multiplying Complex Numbers

Examples: Simplify.

17. $5i(-2+i)$

$-10i + 5i^2$

$-10i + 5(-1)$

$-10i - 5 \longrightarrow \boxed{-5 - 10i}$

18. $(-1+2i)(7-4i)$

$(-1)(7) + (-1)(-4i) + (2i)(7) + (2i)(-4i)$

$-7 \quad + 4i \quad + 14i \quad - 8i^2$

$-7 + 18i - 8(-1) \longrightarrow -7 + 18i + 8 \longrightarrow \boxed{1 + 18i}$

19. $(6+3i)(6-3i)$

$(6)(6) + (6)(-3i) + (3i)(6) + (3i)(-3i)$

$36 \quad \cancel{-18i} \quad \cancel{+18i} \quad -9i^2$

$36 - 9(-1) \longrightarrow 36 + 9 \longrightarrow \boxed{45}$

20. $(2+5i)^2$

$(2+5i)(2+5i)$

$(2)(2) + (2)(5i) + (5i)(2) + (5i)(5i)$

$4 + 10i + 10i + 25i^2$

$4 + 20i + 25(-1)$

$4 + 20i - 25$

$\boxed{-21 + 20i}$

Dividing Complex Numbers

A. Pure Imaginary Denominator

Multiply numerator and denominator by i.

Examples: Simplify.

21. $\dfrac{2+8i}{i} \cdot \dfrac{i}{i} = \dfrac{2i + 8i^2}{i^2} = \dfrac{2i + 8(-1)}{-1} = \dfrac{2i - 8}{-1}$

$\dfrac{2i}{-1} - \dfrac{8}{-1}$

$\boxed{-2i + 8}$

22. $\dfrac{3+7i}{2i} \cdot \dfrac{i}{i} = \dfrac{3i + 7i^2}{2i^2} = \dfrac{3i + 7(-1)}{2(-1)} = \boxed{\dfrac{3i - 7}{-2}}$

$\dfrac{3i}{-2} - \dfrac{7}{-2}$

23. $\dfrac{4-i}{2i} \cdot \dfrac{i}{i} = \dfrac{4i - i^2}{2i^2} = \dfrac{4i - (-1)}{2(-1)} = \boxed{\dfrac{4i + 1}{-2}}$

$\dfrac{4i}{-2} + \dfrac{1}{-2}$