

# RECTANGLES

A rectangle is a quadrilateral with four right angles. It is also a special type of parallelogram (because both pairs of opposite sides are congruent). Thus, a rectangle has all the properties of a parallelogram.

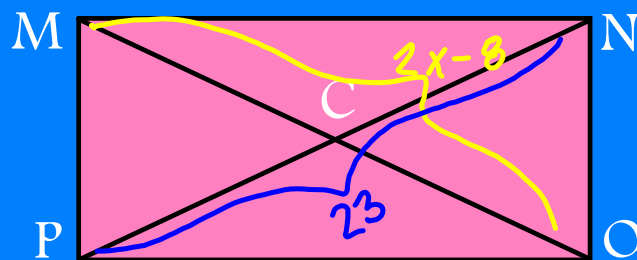
## Theorem 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

### Example 1

Quadrilateral MNOP is a rectangle. Find the value of  $x$  using the given information below.

$$MO = 2x - 8, NP = 23$$

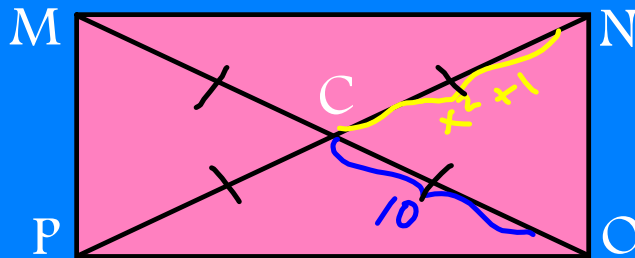


$$\begin{array}{r} 2x - 8 = 23 \\ \quad + 8 \quad + 8 \\ \hline \frac{2x}{2} = \frac{31}{2} \\ \boxed{x = \frac{31}{2}} \end{array}$$

Example 2

Quadrilateral MNOP is a rectangle. Find the value of  $x$  using the given information below.

$$CN = x^2 + 1, CO = 10$$

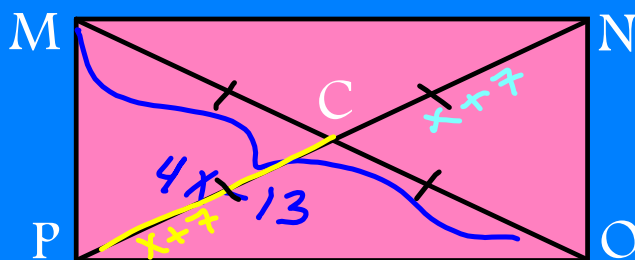


$$\begin{aligned} x^2 + 1 &= 10 \\ -1 \quad -1 \\ \hline \sqrt{x^2} &= \sqrt{9} \\ \boxed{x} &= \pm 3 \end{aligned}$$

Example 3

Quadrilateral MNOP is a rectangle. Find the value of  $x$  using the given information below.

$$MO = 4x - 13, PC = x + 7$$

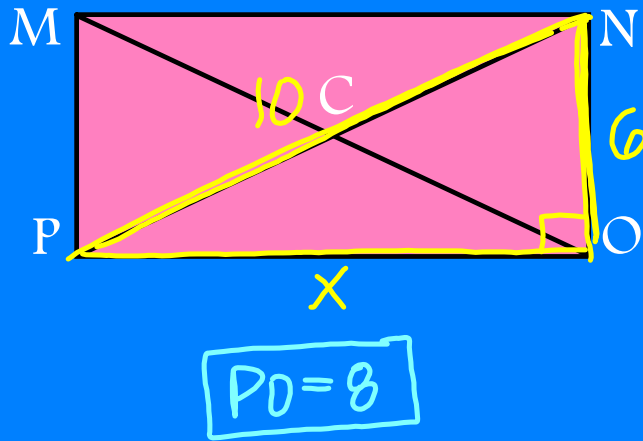


$$\begin{aligned} 4x - 13 &= 2(x + 7) \\ 4x - 13 &= 2x + 14 \\ -2x \quad -2x \\ \hline 2x - 13 &= 14 \\ +13 \quad +13 \\ \hline 2x &= 27 \\ \frac{2x}{2} &= \frac{27}{2} \\ \boxed{x} &= \frac{27}{2} \end{aligned}$$

Example 4

Quadrilateral MNOP is a rectangle. Find PO using the given information below.

$$NO = 6, PN = 10$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + b^2 &= 10^2 \\ 36 + b^2 &= 100 \\ -36 &\quad -36 \\ \hline \sqrt{b^2} &= \sqrt{64} \\ b &= \pm 8 \\ x &= \pm 8 \end{aligned}$$

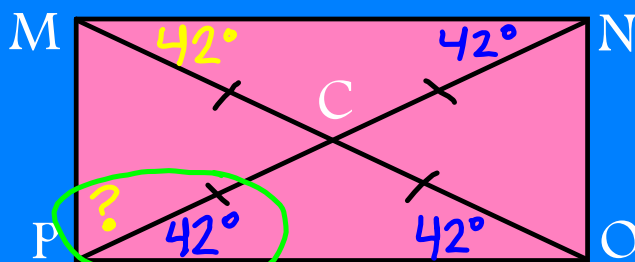
$$PO = 8$$

Example 5

Quadrilateral MNOP is a rectangle.

Find  $m \angle MPN$  using the given information below.

$$m \angle NMO = 42$$



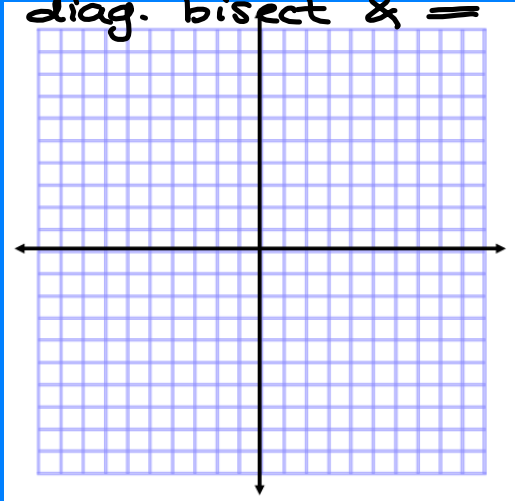
$$\begin{aligned} \text{right } \angle \\ ? + 42 &= 90 \end{aligned}$$

$$m \angle MPN = 48^\circ$$

Example 6

Determine whether quadrilateral ABCD is a rectangle, given A(-6, 9), B(5, 10), C(6, -1), & D(-5, -2).

ABCD is a rect. bc  
diag. bisect & =



diagonals  $\rightarrow$  mdpt

form.



$\rightarrow$  dist. form



$$\begin{array}{l} \text{AC} \\ \left( \frac{-6+6}{2}, \frac{9-1}{2} \right) \text{diag} \\ (0, 4) \end{array} \quad \begin{array}{l} \text{BD} \\ \left( \frac{5-5}{2}, \frac{10-2}{2} \right) \text{diag} \\ (0, 4) \end{array}$$

bisect (Same mdpt)

$$\begin{array}{l} \sqrt{(6+6)^2 + (-1-9)^2} \\ \sqrt{(12)^2 + (-10)^2} \\ \sqrt{144+100} \\ \sqrt{244} \end{array} \quad \text{diag} \quad \cong$$

$$\begin{array}{l} \sqrt{(5-5)^2 + (-2-10)^2} \\ \sqrt{(-10)^2 + (-12)^2} \\ \sqrt{100+144} \\ \sqrt{244} \end{array}$$

If a quadrilateral is a rectangle,  
then the following properties hold true.

1. Opposite sides are congruent & parallel.
2. Opposite angles are supplementary.
3. Consecutive angles are supplementary.
4. Diagonals are congruent and bisect each other.
5. All four angles are right angles.