5.4 Operations with Complex Numbers

Square Root of Negative Numbers

The square root of a negative real number has TWO imaginary roots : one positive, one negative.

$$\sqrt{-r} = \sqrt{-1} \cdot \sqrt{r} \quad \left(\text{where } \sqrt{-1} = i\right) \quad = i\sqrt{r}$$

and

$$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

Examples : Simplify.

1. $\sqrt{-81} = 9i$

3. $\sqrt{-120}$

$\sqrt{\ominus 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}$

$2i\sqrt{30}$

2 | 120
2 | 60
2 | 30
3 | 15
5

2. $\sqrt{-48}$

$\sqrt{\ominus 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$

$4i\sqrt{3}$

2 | 48
2 | 24
2 | 12
2 | 6
3

4. $\sqrt{-256}$

$16i$

Examples: Solve by taking square roots.

5. $x^2 + 16 = 0$

$-16 \quad -16$

$\sqrt{x^2} = \sqrt{-16}$

$x = \pm 4i$

6. $2x^2 + 68 = 20$

$-68 \quad -68$

$\frac{2x^2}{2} = \frac{-48}{2}$

$\sqrt{x^2} = \sqrt{-24}$

$x = \pm 2i\sqrt{6}$

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

7. $-3x^2 - 10 = 44$

$+10 \quad +10$

$\frac{-3x^2}{-3} = \frac{54}{-3}$

$\sqrt{x^2} = \sqrt{-18}$

$x = \pm 3i\sqrt{2}$

$18 = 2 \cdot 3 \cdot 3$

8. $\frac{1}{4}x^2 + 10 = -15$

$-10 \quad -10$

$\frac{\frac{1}{4}x^2}{\frac{1}{4}} = \frac{-25}{\frac{1}{4}}$

$\sqrt{x^2} = \sqrt{-100}$

$x = \pm 10i$

Examples: Solve by taking square roots.

9. $2(x - 1)^2 + 12 = 0$

$-12 \quad -12$

$\frac{2(x-1)^2}{2} = \frac{-12}{2}$

$\sqrt{(x-1)^2} = \sqrt{-6}$

$x - 1 = \pm i\sqrt{6}$

$+1 \quad +1$

$x = 1 \pm i\sqrt{6}$

10. $\frac{1}{2}(x + 4)^2 - 8 = -26$

$+8 \quad +8$

$\frac{2}{1} \cdot \frac{1}{2}(x+4)^2 = -18 \cdot \frac{2}{1}$

$\sqrt{(x+4)^2} = \sqrt{-36}$

$x + 4 = \pm 6i$

$-4 \quad -4$

$x = -4 \pm 6i$

11. $-5(x + 2)^2 - 7 = 38$

$+7 \quad +7$

$\frac{-5(x+2)^2}{-5} = \frac{45}{-5}$

$\sqrt{(x+2)^2} = \sqrt{-9}$

$x + 2 = \pm 3i$

$-2 \quad -2$

$x = -2 \pm 3i$

12. $-\frac{1}{3}(x - 7)^2 + 5 = 23$

$-5 \quad -5$

$\frac{-\frac{1}{3}(x-7)^2}{-\frac{1}{3}} = \frac{18}{-\frac{1}{3}}$

$\sqrt{(x-7)^2} = \sqrt{-54}$

$x - 7 = \pm\sqrt{-2 \cdot 3 \cdot 3 \cdot 3}$

$x - 7 = \pm 3i\sqrt{6}$

$+7 \quad +7$

$x = 7 \pm 3i\sqrt{6}$

$54 = 2 \cdot 3 \cdot 3 \cdot 3$