4.3 Part 2 Logarithmic Functions

We learned about the common logarithm that has base 10. This is the only base that the calculator recognizes.

Example 1

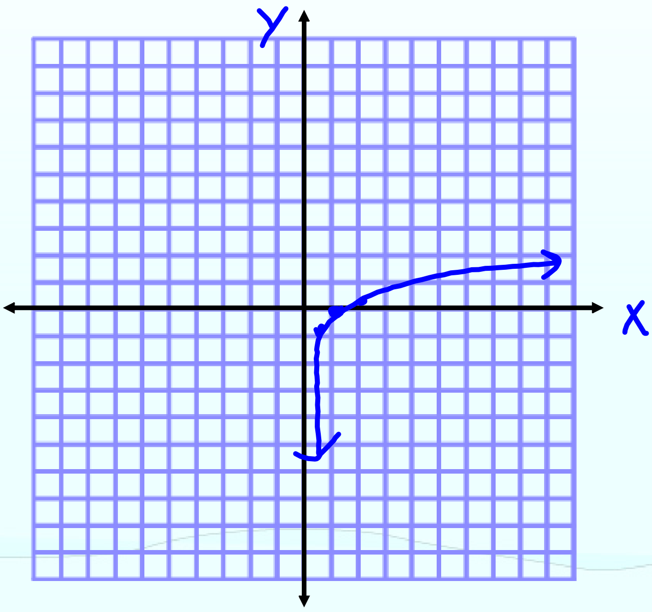
Use a calculator to evaluate each expression to four decimal places.

a) $\log \frac{2}{3} \approx -0.1761$ b) $\log 50 \approx 1.6990$

Example 2

Use a calculator to find appropriate values of $f(x) = \log x$ and use the values to sketch the graph.

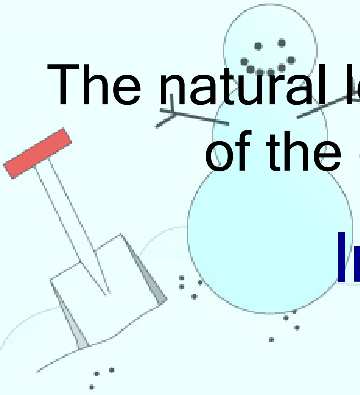
x	y
−2	___
−1	___
0	___
1	0
2	0.301
1/2	−0.301
1/3	−.477



Of all possible bases a for logarithms, it turns out that the most convenient choice for the purposes of calculus is the number e.

The logarithm with the base e is called the natural logarithm.

$$\log_e x = \ln x$$



The natural logarithmic function is the inverse of the exponential function $e^x = y$.

$$\ln x = y \quad \Leftrightarrow \quad e^y = x$$

Example 3

Express in exponential form.

a) $\ln 9 = x$

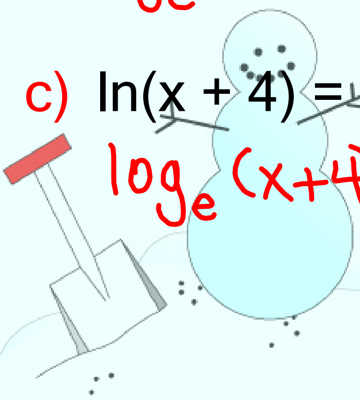
$\log_e 9 = x$

$e^x = 9$

b) $\ln(x - 8) = 4$

$\log_e (x-8) = 4$

$e^4 = x - 8$



c) $\ln(x + 4) = 7$

$\log_e (x+4) = 7$

$e^7 = x + 4$

Example 4

Express in logarithmic form.

a) $e^x = 6$

$\log_e 6 = x$ $\qquad \ln 6 = x$

b) $e^5 = y$

$\log_e y = 5$ $\qquad \ln y = 5$

c) $e^{x-4} = 2.5$

$\log_e 2.5 = x - 4$ $\qquad \ln 2.5 = x - 4$

Properties of Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

Example 5

Evaluate.

a) $\cancel{\ln e}^8 = 8$

b) $\ln \frac{1}{e^2} = \cancel{\ln e}^{-2} = -2$

c) $\cancel{e}^{\ln\sqrt{5}} = \sqrt{5}$

d) $\cancel{e}^{\ln\frac{1}{4}} = \frac{1}{4}$

Example 6

Use a calculator to evaluate to four decimal places.

a) ln 7 ≈ 1.9459

b) ln $\sqrt{6}$ ≈ 0.8959

c) ln $\frac{3}{4}$ ≈ -0.2877

Remember, you cannot take the logarithm of a negative number. This means that you can ONLY take the logarithm of positive numbers. This is important when finding the domain of a logarithm.

Example 7

Find the domain.

a) ln $(4 - x^2)$

must be positive

$4 - x^2 > 0$

$(2-x)(2+x) > 0$

$-\quad +\quad -$

$-3\quad -2\quad 0\quad 2\quad 3$

D: $(-2, 2)$

b) -log x

$x > 0$

D: $(0, \infty)$

c) log (-x)

$\frac{-x}{-1} > \frac{0}{-1}$

$x < 0$

D: $(-\infty, 0)$