4.3 Part 1 Logarithmic Functions

Every exponential function $f(x) = a^x$ with $a > 0$ and $a \neq 1$ is a one-to-one function by the horizontal line test.

Therefore it has an inverse function, which is called the logarithmic function.

exponential form		logarithmic form
$a^{y} = x$	$\Longleftrightarrow$	$\log_a x = y$

exp.

Example 1

Rewrite each exponential in logarithmic form.

a) $10^{5} = 100{,}000$ exp. base $\log_{10} 100{,}000 = 5$

"log base 10 of 100,000 is 5"

b) $3^{2} = 9$ exp. base $\log_3 9 = 2$

c) $\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$ exp. base $\log_{\frac{1}{2}} \frac{1}{4} = 2$

Example 2

Rewrite each logarithm in exponential form.

a) $\log_3 81 = 4$ $\quad 3^4 = 81$

base exp.

b) $\log_2\left(\frac{1}{8}\right) = -3$ $\quad 2^{-3} = \frac{1}{8}$

c) $\log_5 s = r$ $\quad 5^r = s$

It is important to remember
that $\log_a x$ is an exponent!
The answer to every log is the exponent!

Example 3

Evaluate each logarithm.

a) $\log_5 25 = x$ $\quad 5^x = 25 \rightarrow 5^x = 5^2$

$\log_5 25 = 2$

b) $\log_{10} .1 = -1$

$10^x = .1$

$10^x = \frac{1}{10}$

$10^x = 10^{-1}$

c) $\log_{\frac{1}{3}} 27 = -3$

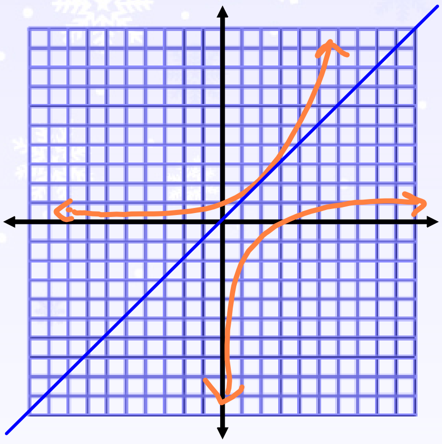
$\frac{1}{3}^x = 27$

$(3^{-1})^x = 3^3$

$\frac{-x}{-1} = \frac{3}{-1}$

$x = -3$

Since the log function is the inverse of the exponential function, it can be graphed by switching the domain and range.



Since $f(x) = a^x$ is a rapidly increasing function, $f(x) = \log_a x$ is a very slowly increasing function.

Notice...

1) that since $a^0 = 1$, then $\log_a 1 = 0$.
2) that since the x-axis is the asymptote for the exponential function, then the y-axis is the asymptote for the log function (unless there is a shift).
3) that the log function is a reflection across the line $y = x$.

Example 4

Sketch the graph by making a table and plotting points. State the domain, range, and any asymptotes.

$f(x) = \log_2 x$

$y = \log_2 x$

$\downarrow$

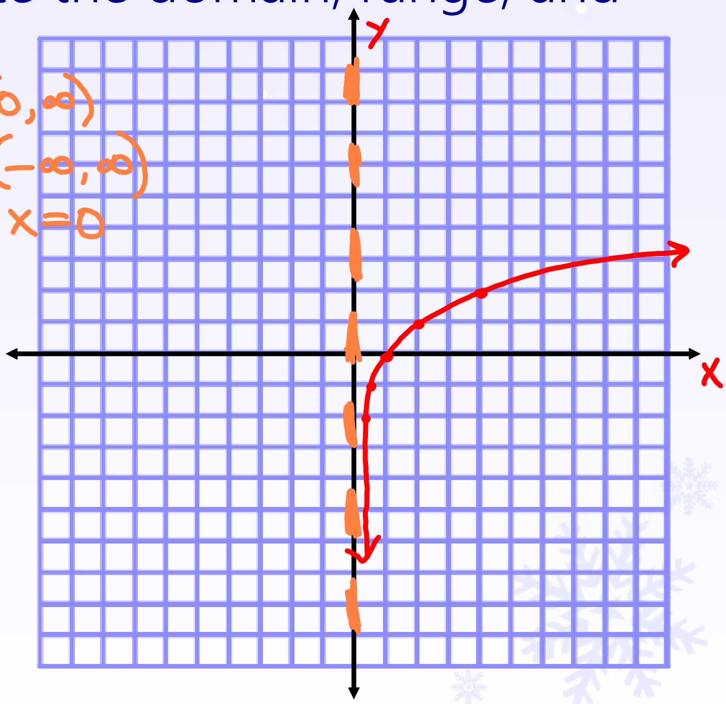
$2^y = x$

x	y
$\frac{1}{4} = 2^{-2}$	-2
$\frac{1}{2} = 2^{-1}$	-1
$1 = 2^0$	0
$2 = 2^1$	1
$4 = 2^2$	2

D: $(0, \infty)$

R: $(-\infty, \infty)$

Asy: $x = 0$



Example 5

Sketch the graph by making a table and plotting points. State the domain, range, and any asymptotes.

$f(x) = 2 + \log_2 (x - 3)$

$y = 2 + \log_2 (x-3)$

$-2 \quad -2$

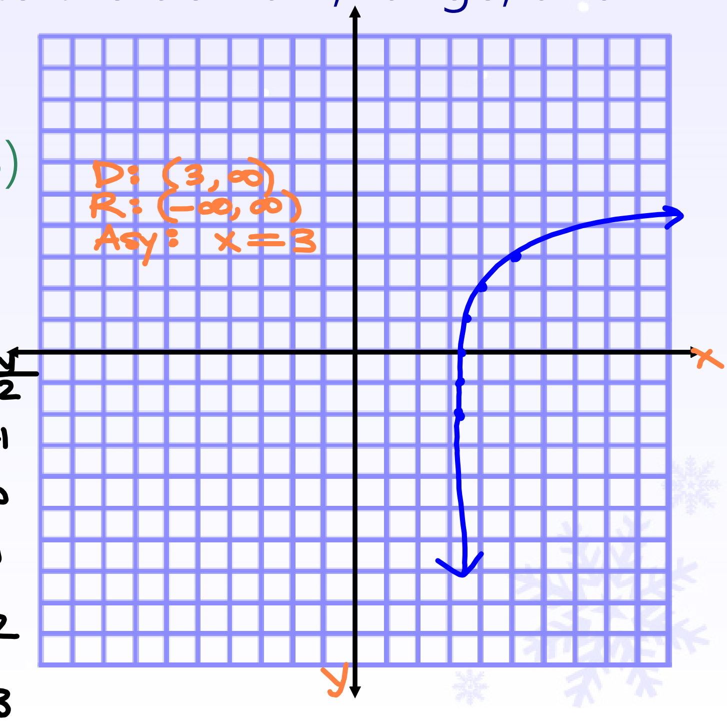
$y - 2 = \log_2 (x-3)$

$2^{y-2} = x - 3$

$+3 \quad +3$

$2^{y-2} + 3 = x$

x	y
$=2^{-4}+3$ $3\frac{1}{16}$	-2
$3\frac{1}{8} = 2^{-3}+3$	-1
$3\frac{1}{4} = 2^{-2}+3$	0
$3\frac{1}{2} = 2^{-1}+3$	1
$4 = 2^{0}+3$	2
$5 = 2^{1}+3$	3



Example 6

Sketch the graph by making a table and plotting points. State the domain, range, and any asymptotes.

$f(x) = \log_2 x$

$f(x) = -\log_2 x$

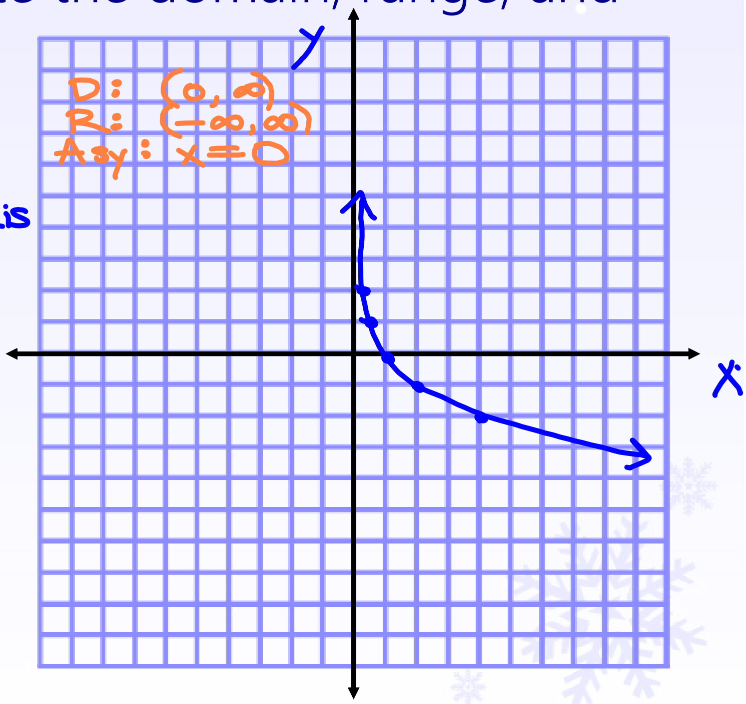
reflection over x-axis

$y = -\log_2 x$

$-y = \log_2 x$

$2^{-y} = x$

x	y
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2



Properties of Logarithms

1. $\log_a 1 = 0 \longrightarrow a^0 = 1$

2. $\cancel{\log_a a} = 1 \longrightarrow a^1 = a$

3. $\cancel{\log_a a}^{x} = x \longrightarrow a^x = a^x$

4. $\cancel{a}^{\cancel{\log_a} x} = x$

The logarithm with base 10 is called the common logarithm and is written by omitting the base.

$$y = \log_{10} x$$

understood to be there

Example 7

Evaluate using the properties of logs.

a) $\cancel{\log_7 7} = 1$

b) $\cancel{8^{\log_8}}\,{}^{11} = 11$

c) $\cancel{\log_2 2}^{5} = 5$

d) $\log_9 1 = 0$

Example 8

Evaluate using the properties of logs.

a) $\log_4 16 = 2$ $\qquad 4^x = 16$ $\qquad \cancel{\log_4} 4^{2}$

b) $\log_2 \frac{1}{16} = -4$ $\qquad \cancel{\log_2} 2^{-4}$

c) $\log_{\frac{1}{3}} 81 = -4$ $\qquad \log_{\frac{1}{3}} \left(\frac{1}{81}\right)^{-1} \rightarrow \cancel{\log_{\frac{1}{3}}} \left(\frac{1}{3}\right)^{-4}$

d) $\log_8 \sqrt{2} = \frac{1}{6}$

$\log_8 2^{1/2} = x \rightarrow 8^x = 2^{1/2} \longrightarrow (2^3)^x = 2^{1/2}$ $\qquad \frac{1}{3} \cdot 3x = \frac{1}{2} \cdot \frac{1}{3}$

$x = \frac{1}{6}$

Example 9

Evaluate using the properties of logs.

a) $\log_5 \frac{1}{125} = -3$ $\qquad \log_5 \frac{1}{5^3} \rightarrow \cancel{\log_5} 5^{-3}$

b) $\log_{\frac{1}{4}} 2 = -\frac{1}{2}$ $\qquad \frac{1}{4}^x = 2 \rightarrow \left(\frac{1}{2^2}\right)^x = 2 \rightarrow (2^{-2})^{x} = 2^1$

$-2x = 1$

c) $\log_{25} \sqrt[4]{5} = \frac{1}{8}$ $\qquad 25^x = 5^{\frac{1}{4}} \rightarrow (5^2)^x = 5^{\frac{1}{4}}$

$2x = \frac{1}{4}$

$x = \frac{1}{8}$

d) $\log_{10} 1000 = 3$ $\qquad \cancel{\log_{10}} 10^{3}$