6.3 TESTS FOR PARALLELOGRAMS

**Theorem 6.6**
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Example 1**
The coordinates of the vertices of quadrilateral PQRS are P(-5, 3), Q(-1, 5), R(6, 1), & S(2, -1). Determine if quadrilateral PQRS is a parallelogram using the above theorem. Explain your work.

\[
PQ = \sqrt{(-1+5)^2 + (5-3)^2} = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}
\]

\[
QR = \sqrt{(6+1)^2 + (-5-5)^2} = \sqrt{7^2 + (-10)^2} = \sqrt{49 + 100} = \sqrt{149} = \sqrt{149}
\]

\[
RS = \sqrt{(2-6)^2 + (-1-1)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}
\]

\[
PS = \sqrt{(2+5)^2 + (-3-3)^2} = \sqrt{7^2 + (-6)^2} = \sqrt{49 + 36} = \sqrt{85}
\]

PQRS is a \(\square\) b/c opp. sides \(\cong\)

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**Theorem 6.7**
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

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**Theorem 6.8**
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.
Theorem 6.9
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Example 2
The coordinates of the vertices of quadrilateral ABCD are A(−1, 3), B(2, 1), C(9, 2), & D(6, 4). Determine if quadrilateral ABCD is a parallelogram using the above theorem. Explain your work.

\[
\begin{align*}
AC &= \left(\frac{-1+9}{2}, \frac{3+2}{2}\right) \\
&= \left(4, \frac{5}{2}\right) \\
\text{BD} &= \left(\frac{2+6}{2}, \frac{1+4}{2}\right) \\
&= \left(4, \frac{5}{2}\right)
\end{align*}
\]

\[
\begin{align*}
\text{Diag. share a midpt} \\
\therefore \text{ABCD is } \square
\end{align*}
\]

Theorem 6.10
If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Example 3
The coordinates of the vertices of quadrilateral ABCD are A(−1, 3), B(2, 1), C(9, 2), & D(6, 4). Determine if quadrilateral ABCD is a parallelogram using the above theorem. Explain your work.

\[
\begin{align*}
\text{AB} &= \sqrt{(2+1)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13} \\
\text{CD} &= \sqrt{(6-9)^2 + (4-2)^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13}
\end{align*}
\]

\[
\begin{align*}
\text{m} &= \frac{1-3}{2+1} = \frac{-2}{3} \parallel \text{m} = \frac{4-2}{6-9} = \frac{-2}{-3} \\
\therefore \text{ABCD is } \square
\end{align*}
\]
Example 4
The coordinates of the vertices of quadrilateral EFGH are E(6, 5), F(6, 11), G(14, 18), & H(14, 12). Determine if quadrilateral EFGH is a parallelogram using one of the following theorems: Theorem 6.6, Theorem 6.9, or Theorem 6.10. Explain your work.

A quadrilateral is a parallelogram if any one of the following is true...

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 6.6)
3. Both pairs of opposite angles are congruent. (Theorem 6.7)
4. An angle is supplementary to both consecutive angles. (Theorem 6.8)
5. Diagonals bisect each other. (Theorem 6.9)
6. A pair of opposite sides is both parallel and congruent. (Theorem 6.10)