**6.1 Polygons**

**POLYGON** - A figure in a plane that meets the following conditions:
1. It is a closed figure formed by 3 or more coplanar segments called sides.
2. Sides that have a common endpoint are noncollinear.
3. Each side intersects exactly two other sides, but only at their endpoints.

Which of the following are polygons?

- [ ]  
- [X]  
- [ ]  
- [ ]  
- [ ]  
- [ ]  
- [X]  
- [ ]  
- [ ]  
- [ ]
Example 1
Decide whether the figures below are polygons. If it is not, explain why.

- a) not a segment
- b) yes
- c) no Intersect at more than endpoints
- d) not closed
- e) yes
- f) yes

A convex polygon is a polygon such that no line containing a side of the polygon contains a point in the interior of the polygon.

A polygon that is not convex is concave.
Polygons are classified by the number of sides they have.

<table>
<thead>
<tr>
<th># of Sides</th>
<th>Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>triangle</td>
</tr>
<tr>
<td>4</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>pentagon</td>
</tr>
<tr>
<td>6</td>
<td>hexagon</td>
</tr>
<tr>
<td>7</td>
<td>heptagon</td>
</tr>
<tr>
<td>8</td>
<td>octagon</td>
</tr>
<tr>
<td>9</td>
<td>nonagon</td>
</tr>
<tr>
<td>10</td>
<td>decagon</td>
</tr>
<tr>
<td>12</td>
<td>dodecagon</td>
</tr>
<tr>
<td>n</td>
<td>n-gon</td>
</tr>
</tbody>
</table>

When referring to a polygon, we use its name and list the vertices in consecutive order.

Pentagon RSTUV and pentagon TUVRS are two possible correct names for the polygon at the left.

The polygon above is a regular polygon, which means it is a convex polygon with all sides congruent and all angles congruent.
Polygons with more than 3 sides have diagonals.

**diagonal** - joins 2 nonadjacent vertices

Theorem 6.1: Interior Angles of a Quadrilateral
The sum of the measures of the interior angles of a quadrilateral is $360^\circ$.

**Example 2**
Find the measure of each missing angle.

a) \[110 + 100 + 60 + m\angle A = 360^\circ\]
\[270 + m\angle A = 360^\circ\]
\[m\angle A = 90^\circ\]

b) \[55 + 80 + 160 + m\angle A = 360^\circ\]
\[295 + m\angle A = 360^\circ\]
\[m\angle A = 65^\circ\]
Example 3  
Find $m \angle Q$ and $m \angle R$.

\[
\begin{align*}
80 + 70 + x + 2x &= 360 \\
150 + 3x &= 360 \\
-150 &= -150 \\
3x &= 210 \\
\frac{3x}{3} &= \frac{210}{3} \\
x &= 70 \\
\end{align*}
\]

$m \angle Q = 70^\circ$

$m \angle R = 140^\circ$

Example 4  
Find $m \angle F$, $m \angle G$ and $m \angle H$.

\[
\begin{align*}
55 + 55 + x + x &= 360 \\
110 + 2x &= 360 \\
-110 &= -110 \\
2x &= 250 \\
\frac{2x}{2} &= \frac{250}{2} \\
x &= 125 \\
\end{align*}
\]

$m \angle F = 125^\circ$

$m \angle G = 55^\circ$

$m \angle H = 125^\circ$
Example 5
Find $m \angle K$, $m \angle L$ and $m \angle M$.

\[ 80 + x + x + (x - 20) = 360 \]
\[ 3x + 60 = 360 \]
\[ 3x = 300 \]
\[ x = 100 \]