# 8.3 TESTS FOR PARALLELOGRAMS

## Theorem 8.7

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

## Example 1

The coordinates of the vertices of quadrilateral PQRS are P(-5, 3), Q(-1, 5), R(6, 1), & S(2, -1). Determine if quadrilateral PQRS is a parallelogram using the **above theorem.** Explain your work.

$PQ = \sqrt{(-1+5)^2+(5-3)^2}$
$= \sqrt{(4)^2+(2)^2}$
$= \sqrt{16+4}$
$= \sqrt{20} \rightarrow 2\sqrt{5}$

$RS = \sqrt{(2-6)^2+(-1-1)^2}$
$= \sqrt{(-4)^2+(-2)^2}$
$= \sqrt{16+4}$
$= \sqrt{20} \rightarrow 2\sqrt{5}$

$PQ = RS$ one pair opp. sides $\cong$

$QR = \sqrt{(6+1)^2+(1-5)^2}$
$= \sqrt{(7)^2+(-4)^2}$
$= \sqrt{49+16}$
$= \sqrt{65}$

$SP = \sqrt{(2+5)^2+(-1-3)^2}$
$= \sqrt{(7)^2+(-4)^2}$
$= \sqrt{49+16}$
$= \sqrt{65}$

$QR = SP$ second pair opp sides $\cong$

PQRS is a ▱ b/c both pairs opp sides $\cong$

**Theorem 8.8**

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

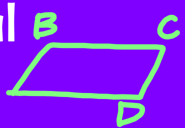
**Theorem 8.9**

If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

slope form. dist. form.

**Example 2**

The coordinates of the vertices of quadrilateral ABCD are A(-1, 3), B(2, 1), C(9, 2), & D(6, 4). Determine if quadrilateral ABCD is a parallelogram using the **above** theorem. Explain your work.




AB

$\sqrt{(2+1)^2+(1-3)^2}$

$\sqrt{(3)^2+(-2)^2}$

$\sqrt{9+4}$

$\sqrt{13}$

$m=\frac{1-3}{2+1}=-\frac{2}{3}$

ABCD is ▱ b/c opp. sides ≅ & //

$\overline{AB}\cong\overline{CD}$

$\overline{AB}\parallel\overline{CD}$

CD

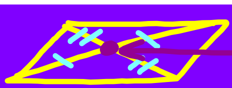
$\sqrt{(6-9)^2+(4-2)^2}$

$\sqrt{(-3)^2+(2)^2}$

$\sqrt{9+4}$

$\sqrt{13}$

$m=\frac{4-2}{6-9}=-\frac{2}{3}$



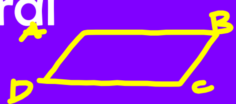
midpoint of each diagonal is same

**Theorem 8.10**

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**Example 3**

The coordinates of the vertices of quadrilateral ABCD are A(-1, 3), B(2, 1), C(9, 2), & D(6, 4). Determine if quadrilateral ABCD is a parallelogram using the **above** theorem. Explain your work.




AC

$$\left(\frac{-1+9}{2}, \frac{3+2}{2}\right)$$

$$\left(\frac{8}{2}, \frac{5}{2}\right)$$

$$\left(4, \frac{5}{2}\right)$$

$\overline{AC}$ & $\overline{BD}$ have same mdpt

↓

diag. bisect

↓

BD

$$\left(\frac{2+6}{2}, \frac{1+4}{2}\right)$$

$$\left(\frac{8}{2}, \frac{5}{2}\right)$$

$$\left(4, \frac{5}{2}\right)$$

**Example 4**

The coordinates of the vertices of quadrilateral EFGH are E(6, 5), F(6, 11), G(14, 18), & H(14, 12). Determine if quadrilateral EFGH is a parallelogram using one of the following theorems: Theorem 8.7, Theorem 8.9, **or** Theorem 8.10. Explain your work.

EG

$$\left(\frac{6+14}{2}, \frac{5+18}{2}\right)$$

$$\left(10, \frac{23}{2}\right)$$

FH

$$\left(\frac{6+14}{2}, \frac{11+12}{2}\right)$$

$$\left(10, \frac{23}{2}\right)$$

$\overline{EG}$ & $\overline{FH}$ same mdpt

↓

diag. bisect

↓

EFGH is ▱

# A quadrilateral is a parallelogram if any one of the following is true...

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 8.7)
3. Both pairs of opposite angles are congruent. (Theorem 8.8)
4. An angle is supplementary to both consecutive angles.
5. A pair of opposite sides is both parallel and congruent. (Theorem 8.9)
6. Diagonals bisect each other. (Theorem 8.10)