5.3 Solve Quadratics by Square Roots

I. Square Root of Positive Number

A number r is a square root of s if $r^2 = s$

$\sqrt{s} = r$ iff $r^2 = s$

radical sign

radicand (number under the radical sign)

A positive number has TWO square roots: $\sqrt{s}$ and $-\sqrt{s}$

$\sqrt{100} = 10$, since $10^2 = 100$

$\sqrt{100} = -10$, since $(-10)^2 = 100$

To simplify a radical (if you do not know the root/answer), factor the radicand using prime factors.

prime factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc.

$\sqrt{\#} \leftarrow$ radicand

EXAMPLES:

196=radicand

1. $\sqrt{196}$

$$\begin{array}{r|l} 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ & 7 \end{array}$$

$$\sqrt{\boxed{2 \cdot 2} \cdot \boxed{7 \cdot 7}}$$

$$2 \cdot 7 = 14$$

2. $\sqrt{80}$

$$\begin{array}{r|l} 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

$$\sqrt{\boxed{2 \cdot 2} \cdot \boxed{2 \cdot 2} \cdot 5}$$

$$2 \cdot 2$$

$$\boxed{4\sqrt{5}}$$

KEY CONCEPT *For Your Notebook*

Properties of Square Roots ($a > 0, b > 0$)

Product Property $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ **Example** $\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

Quotient Property $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ **Example** $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

A square root is simplified when:
1) the radicand has NO perfect square factors other than 1,
2) the radicand is NOT a fraction, and
3) NO radical is in the denominator. bottom

3. $\sqrt{(x+5)^2}$

$\sqrt{(x+5)(x+5)}$

$\boxed{x+5}$

6. $\sqrt{(x-4)^2}$

$\sqrt{(x-4)(x-4)}$

$\boxed{x-4}$

4. $\sqrt{3} \bullet \sqrt{75} = \sqrt{3 \cdot 75}$

$= \sqrt{225}$

$\sqrt{3 \cdot 3 \cdot 5 \cdot 5}$

$3 \cdot 5 = \boxed{15}$

$$\begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

7. $\sqrt{6} \bullet \sqrt{21}$

$\sqrt{2 \cdot 3 \cdot 3 \cdot 7}$

$\boxed{3\sqrt{14}}$

5. $\sqrt{\frac{4}{81}}$ $\frac{\sqrt{4}}{\sqrt{81}} = \boxed{\frac{2}{9}}$

8. $\frac{\sqrt{7}}{\sqrt{16}} = \boxed{\frac{\sqrt{7}}{4}}$

II. Rationalize Denominator Containing Square Root

To rationalize transforms a fraction to an equivalent form with NO radical in the denominator.

Steps:

1. Reduce the fraction, if possible.
   Reduce like parts: radicand to radicand; coefficient to coefficient
2. Multiply top and bottom by the square root in the denominator.
3. Simplify top and bottom. Reduce again, if possible.

EXAMPLES:

9. $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\boxed{\frac{1\sqrt{2}}{2}}$

11. $\frac{\sqrt{12} \div 6}{\sqrt{18} \div 6} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\boxed{\frac{\sqrt{6}}{3}}$

10. $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3} \div 3}{3 \div 3}$

$\frac{2\sqrt{3}}{1}$

$\boxed{2\sqrt{3}}$

12. $\frac{\sqrt{64}}{\sqrt{16}} = \frac{8}{4} = \boxed{2}$

III. Solve Quadratic Equation By Square Roots

When solving a quadratic equation by square roots, you are finding both the positive and negative roots.

If I $\sqrt{\ }$ both sides of an equation,

$$\boxed{\text{If } x^2 = a \text{ then } x = \pm\sqrt{a}.}$$

then I must include $\pm$ in my answer!

NOTE: To solve by square roots, you must isolate x^2 on one side of the equation.

A. Form $ax^2 = c$ or $ax^2 - c = 0$ (no bx term)

$(\ )^2$ & $\sqrt{\ }$ $\Rightarrow$ inverse operations

EXAMPLES:

13. $4\boxed{x^2} = 240$

$$\frac{4x^2}{4} = \frac{240}{4}$$

$$\cancel{\sqrt{x^2}} = \sqrt{60}$$

$$x = \pm\sqrt{2\cdot 2\cdot 3\cdot 5}$$

$2 \,|\, 60$
$2 \,|\, 30$
$3 \,|\, 15$
5

$$\boxed{x = \pm 2\sqrt{15}}$$

14. $2\boxed{x^2} + 7 = 88$

$$\quad -7 \quad -7$$

$$\frac{2\boxed{x^2}}{2} = \frac{81}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{81}{2}}$$

$$x = \pm\frac{9}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{x = \pm\frac{9\sqrt{2}}{2}}$$

B. Form $a(x - h)^2 + k = 0$ or $a(x - h)^2 = k$.
Still solving for x!!!

Isolate parentheses. Take square root of both sides. Then add or subtract to isolate x.

EXAMPLES:

15. $(x + 3)^2 - 25 = 0$

$+25 \quad +25$

$\sqrt{(x+3)^2} = \sqrt{25}$

$x + 3 = \pm 5$

$-3 \quad -3$

$x = -3 \pm 5$

$-3 + 5 \qquad -3 - 5$

$\boxed{x = 2} \qquad \boxed{x = -8}$

16. $-2(x - 1)^2 = -12$

$\div -2 \quad \div -2$

$\sqrt{(x-1)^2} = \sqrt{6}$

$2 | 6$, 3

$x - 1 = \pm\sqrt{6}$

$+1 \quad +1$

$\boxed{x = 1 \pm \sqrt{6}}$

$x = 1 + \sqrt{6} \qquad x = 1 - \sqrt{6}$

17. $\frac{1}{3}(x - 2)^2 - 4 = 11$

$+4 \quad +4$

$\frac{\frac{1}{3}(x-2)^2}{\frac{1}{3}} = \frac{15}{\frac{1}{3}}$

$\sqrt{(x-2)^2} = \sqrt{45}$

$5 | 45$, $3 | 9$, 3

$x - 2 = \pm\sqrt{5 \cdot 3 \cdot 3}$

$x - 2 = \pm 3\sqrt{5}$

$+2 \quad +2$

$\boxed{x = 2 \pm 3\sqrt{5}}$

18. $5(x + 3)^2 + 9 = 20$

$-9 \quad -9$

$\frac{5(x+3)^2}{5} = \frac{11}{5}$

$\sqrt{(x+3)^2} = \sqrt{\frac{11}{5}}$

$\sqrt{\frac{11}{5}} = \frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{55}}{5}$

$x + 3 = \pm\frac{\sqrt{55}}{5}$

$-3 \quad -3$

$\boxed{x = -3 \pm \frac{\sqrt{55}}{5}}$

Attachments

PRACTICE WORKSHEET Square Roots and Quad Equations.doc

Worksheet Simplify Square Roots.doc