4.2 The Natural Exponential Function

Although any positive number can be used for the base, the most important base is the number denoted by e.

e is defined as the value that $(1 + \frac{1}{n})^n$ approaches as n becomes large (in calculus, this idea will be made more precise).

$e \approx 2.718281828459045235 36...$

The natural exponential function is $f(x) = e^x$.

This is also often referred to as *the* exponential function.

Evaluate to five decimal places.

$e^3 \approx 20.08554$

$2e^{-.53} \approx 1.17721$

$e^{4.8} \approx 121.51042$

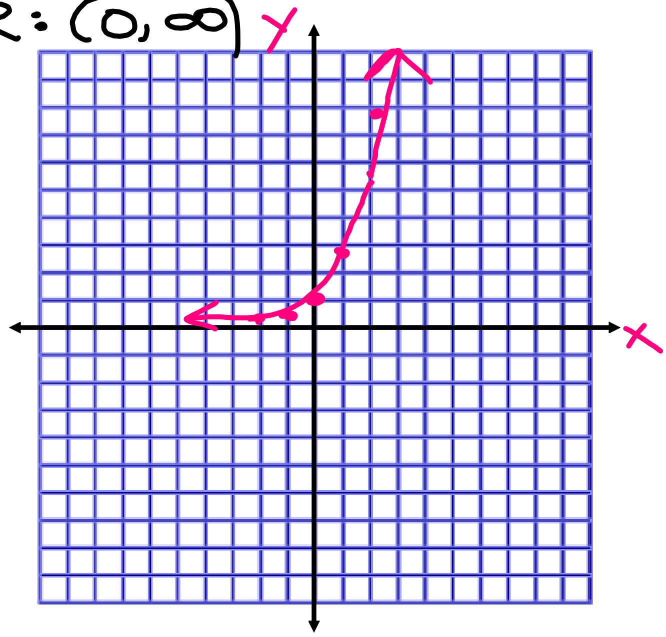
Sketch the graph of the functions.

State the domain and range.

D: $(-\infty, \infty)$ R: $(0, \infty)$

$f(x) = e^x$

x	y
-2	.135
-1	.368
0	1
1	2.718
2	7.389



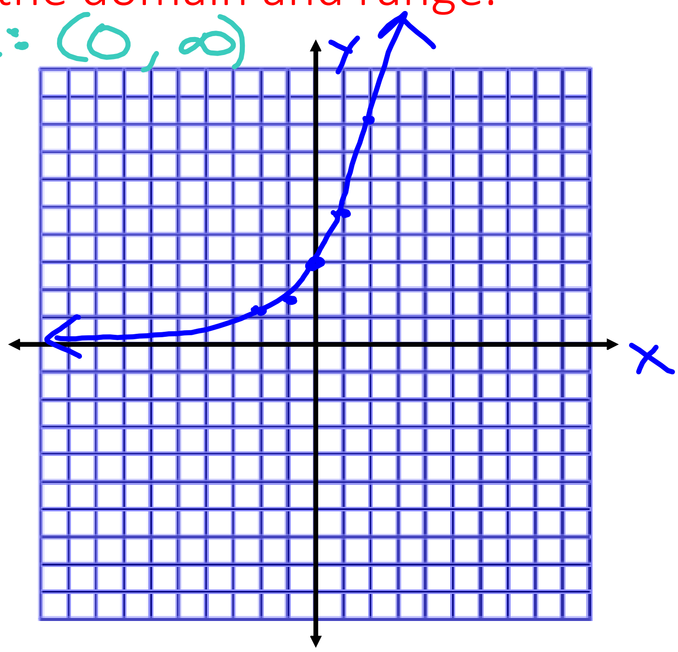
Sketch the graph of the functions.

State the domain and range.

D: $(-\infty, \infty)$ R: $(0, \infty)$

$g(x) = 3e^{.5x}$ $+0$

x	y
-2	1.104
-1	1.820
0	3
1	4.946
2	8.155



Continuous Compound Interest

$$A(t) = Pe^{rt}$$

$A(t)$ = the amount of \$ after t years

P = the amount of \$ invested or borrowed

$e = e$

r = the percent as a decimal

t = the number of years

Example $P = 1000$ $r = .12$

Find the amount of interest if \$1000 is invested at a rate of 12% per year, compounded continuously. 4 years $= t$

$A = 1000e^{(.12)(4)}$

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Example

A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function $m(t) = 13e^{-0.015t}$ where $m(t)$ is measured in kg.

a) Find the mass at time $t = 0$.

$m(0) = 13$ kg

b) How much of the mass remains after 45 days?

$m(45) \approx 6.619$ kg