

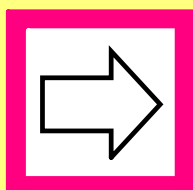
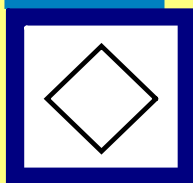
## 8.1 Find Angle Measures in Polygons

**POLYGON** - A figure in a plane that meets the following conditions:

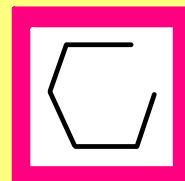
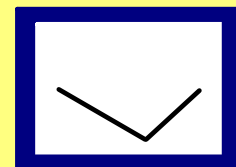
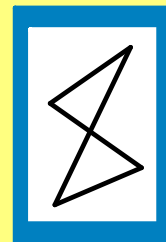
1. It is a closed figure formed by 3 or more coplanar segments called sides.
2. Sides that have a common endpoint are noncollinear.
3. Each side intersects exactly two other sides, but only at their endpoints.

Which of the following are polygons?

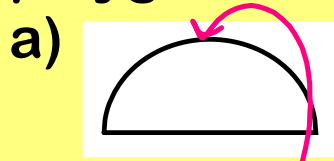
P  
O  
L  
Y  
G  
O  
N



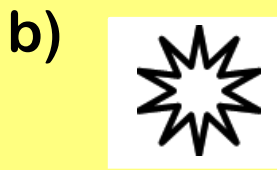
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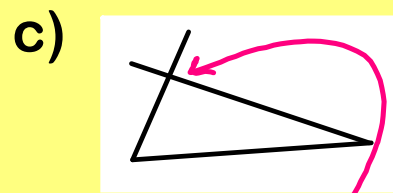
Decide whether the figures below are polygons. If it is not, explain why.



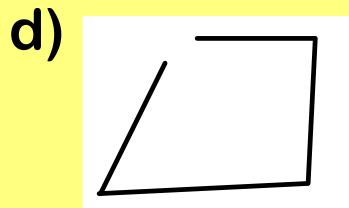
no  
not a segment



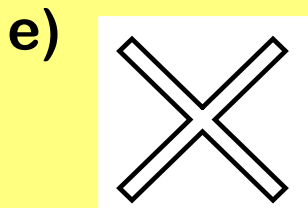
yes



no  
doesn't  
intersect  
at endpoints



no  
not closed

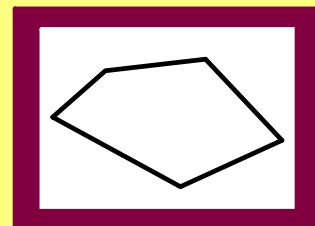
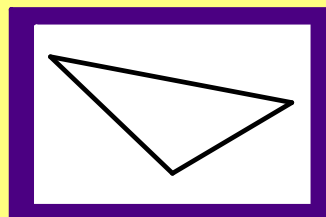
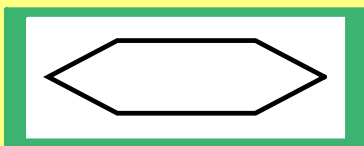


yes

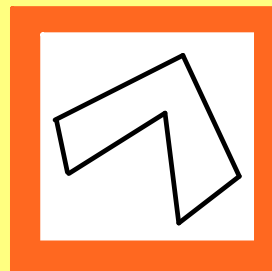
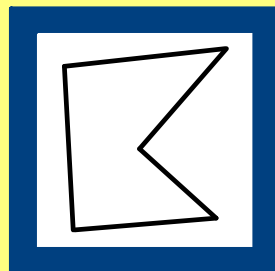


yes

A convex polygon is a polygon such that no line containing a side of the polygon contains a point in the interior of the polygon.

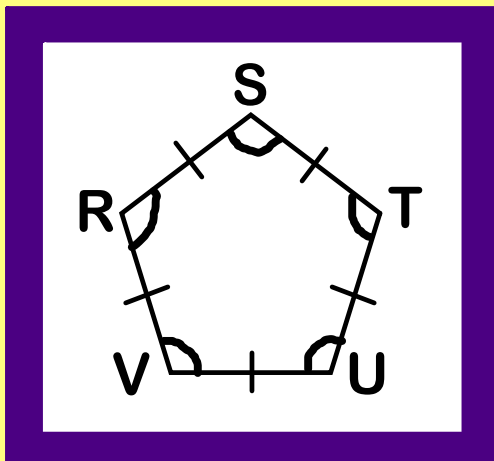


A polygon that is not convex is concave.



**Polygons are classified by the number of sides they have.**

# of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon



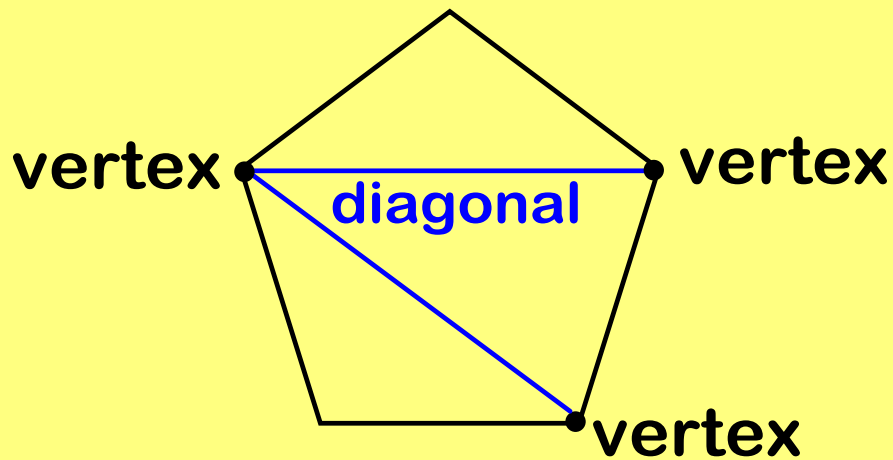
When referring to a polygon, we use its name and list the vertices in consecutive order.

Pentagon RSTUV and pentagon TUVRS are two possible correct names for the polygon at the left.

The polygon above is a regular polygon, which means it is a convex polygon with all sides congruent and all angles congruent.

Polygons with more than 3 sides have diagonals.

diagonal - joins 2 nonadjacent vertices



### Theorem 8.1: Polygon Interior Angle Theorem

If a convex polygon has  $n$  sides and  $S$  is the sum of the measures of its interior angles, then  $S = 180(n - 2)$ .

#### Example 1

Find the measure of each interior angle of the largest pentagon-shaped section of the Pentagon building. *regular*

$$n = 5$$

$$S = 180(5 - 2)$$

$$S = 180(3)$$

$$S = 540^\circ$$

$$\frac{540^\circ}{5} =$$

$$108^\circ = \text{ea. int. } \angle$$

Example 2

Six angles of a convex polygon are congruent. Each of the other two angles has a measure of 20 more than the measure of each of the other six angles. Find the measure of each angle.

$$n=8$$

$$S = 180(8-2)$$

$$S = 180(6)$$

$$S = 1080^\circ$$

$$6 \angle's = 130^\circ$$

$$2 \angle's = 150^\circ$$

$$6x + 2(x+20) = 1080$$

$$6x + 2x + 40 = 1080$$

$$8x + 40 = 1080$$

$$\frac{8x}{8} = \frac{1040}{8}$$

$$x = 130$$

Example 3

The sum of the measures of the interior angles of a convex polygon is  $2340^\circ$ . Classify the polygon by the number of sides.  $n=?$

$$S = 180(n-2)$$

$$2340 = 180(n-2)$$

$$2340 = 180n - 360$$

$$\begin{array}{r} 2340 \\ +360 \\ \hline 2700 \end{array} = \begin{array}{r} 180n \\ +360 \\ \hline 180n \end{array}$$

$$\frac{2700}{180} = \frac{180n}{180}$$

$$n = 15$$

15-gon

**Example 4**Use the diagram below to find  $m\angle S$  and  $m\angle T$ .

6 sides  
 $S = 180(6-2)$   
 $S = 720$

$m\angle S = 110^\circ$   
 $m\angle T = 110^\circ$

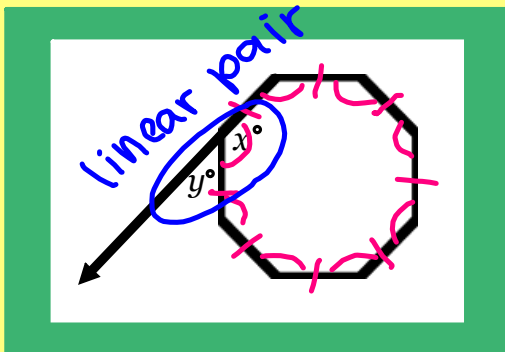
$$95 + 150 + 165 + 90 + 2x = 720$$

$$500 + 2x = 720$$

$$2x = 220$$

$$x = 110$$
**Theorem 8.2: Polygon Exterior Angle Theorem**

If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

**Example 5**Find the measure of each interior angle and exterior angle of a regular octagon.

$$\frac{360}{8} \text{ (all ext } \angle\text{s)}$$

$$45^\circ = \text{ext } \angle = y$$

$$135^\circ = \text{int } \angle = x$$

**Example 6**

The measure of an exterior angle of a regular polygon is 14.4.

Find the number of sides in the polygon.

$$\frac{360}{14.4} \text{ (all ext } \angle\text{s)} = 25$$

25 sides

**Example 7**

The measure of an interior angle of a regular polygon is 144.

Find the number of sides in the polygon.

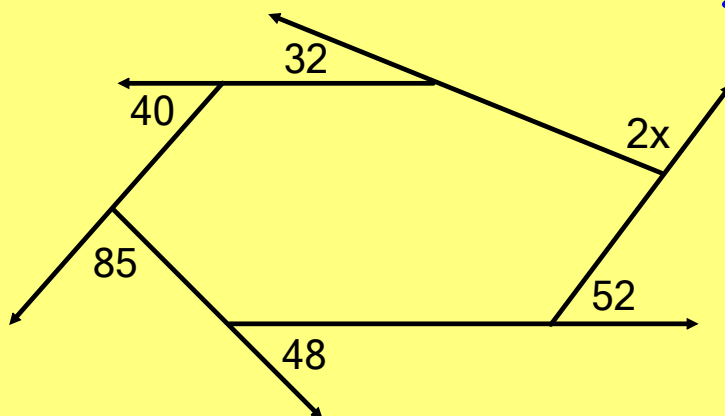
$$\text{ext } \angle = 36^\circ$$

$$\frac{360}{36} = 10$$

10 sides

**Example 8**

Find the value of x.



$$32 + 40 + 85 + 48 + 52 + 2x = 360$$

$$257 + 2x = 360$$

$$2x = 103$$

$$x = 51.5$$